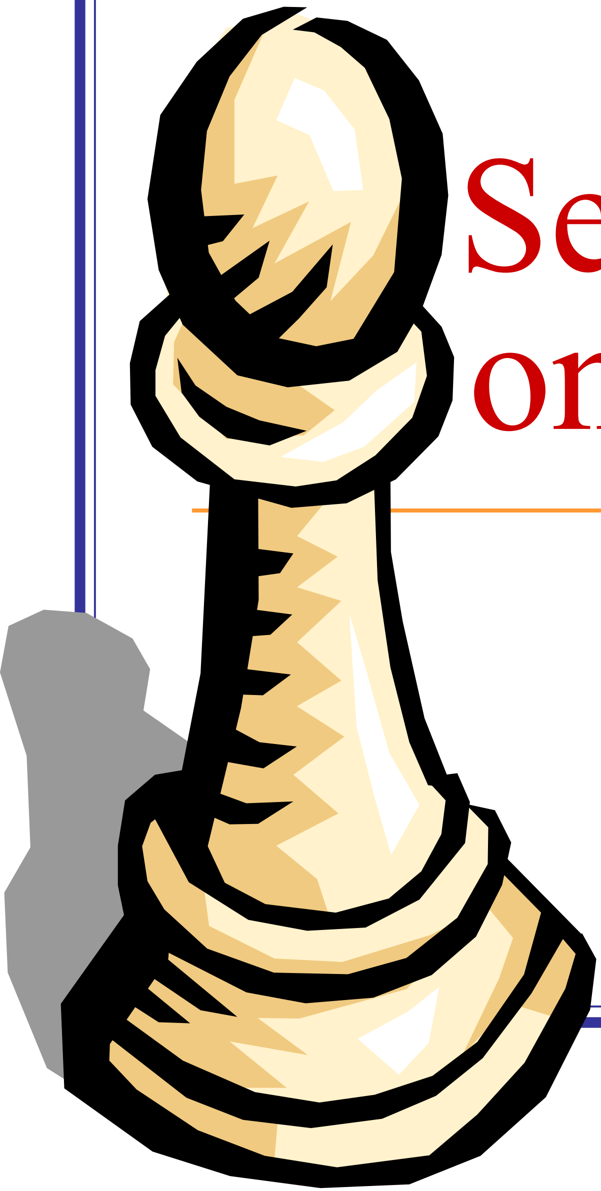

Separating Queens on the Chessboard

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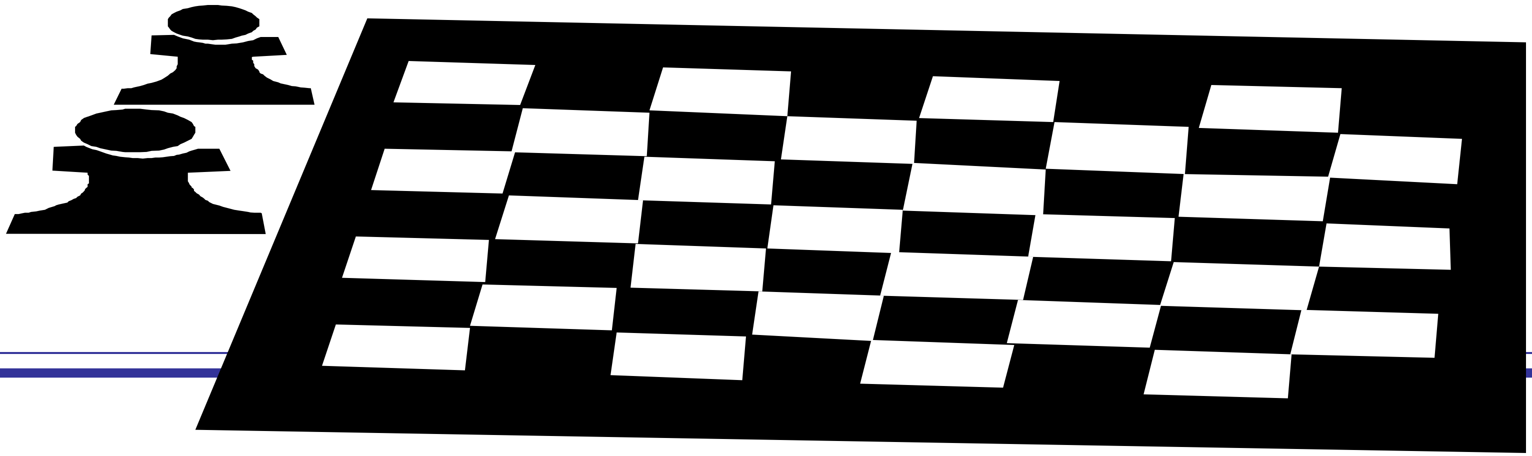
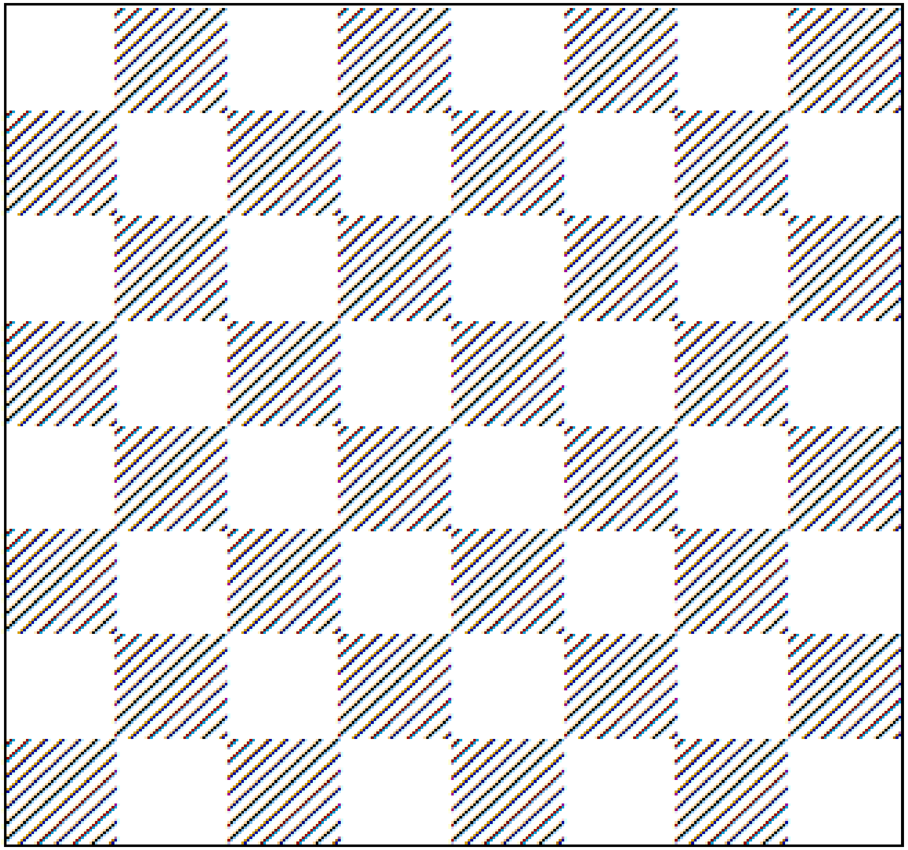


Chessboard Graphs

- Queens Graph
- Rooks Graph
- Bishops Graph

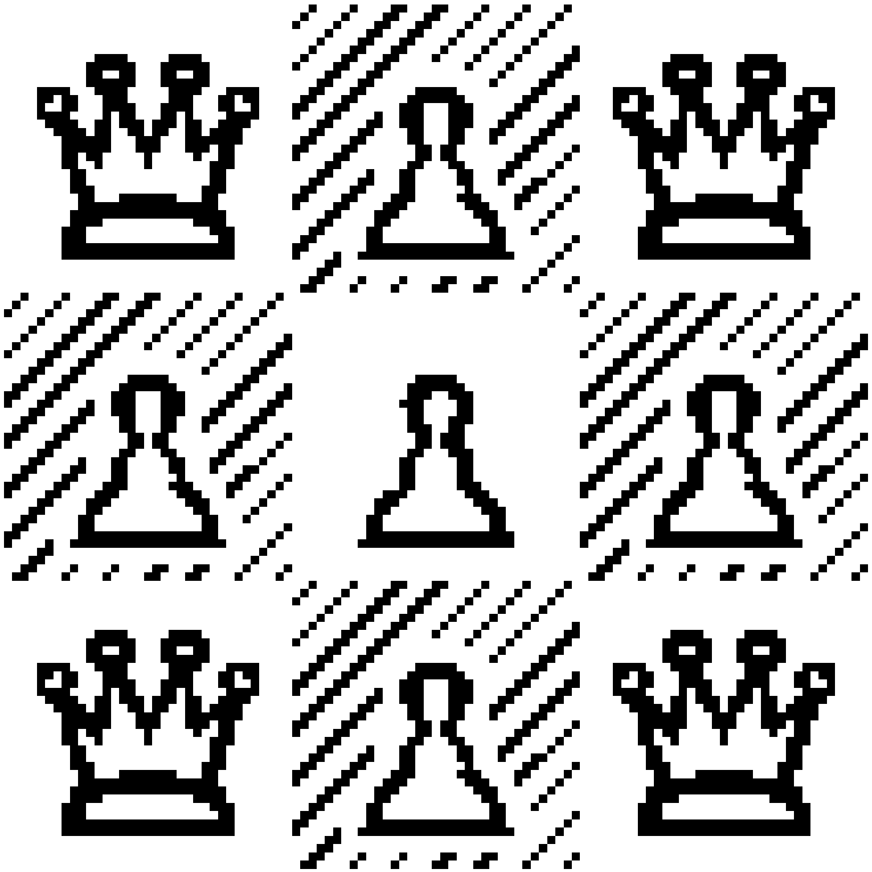


π-separation numbers

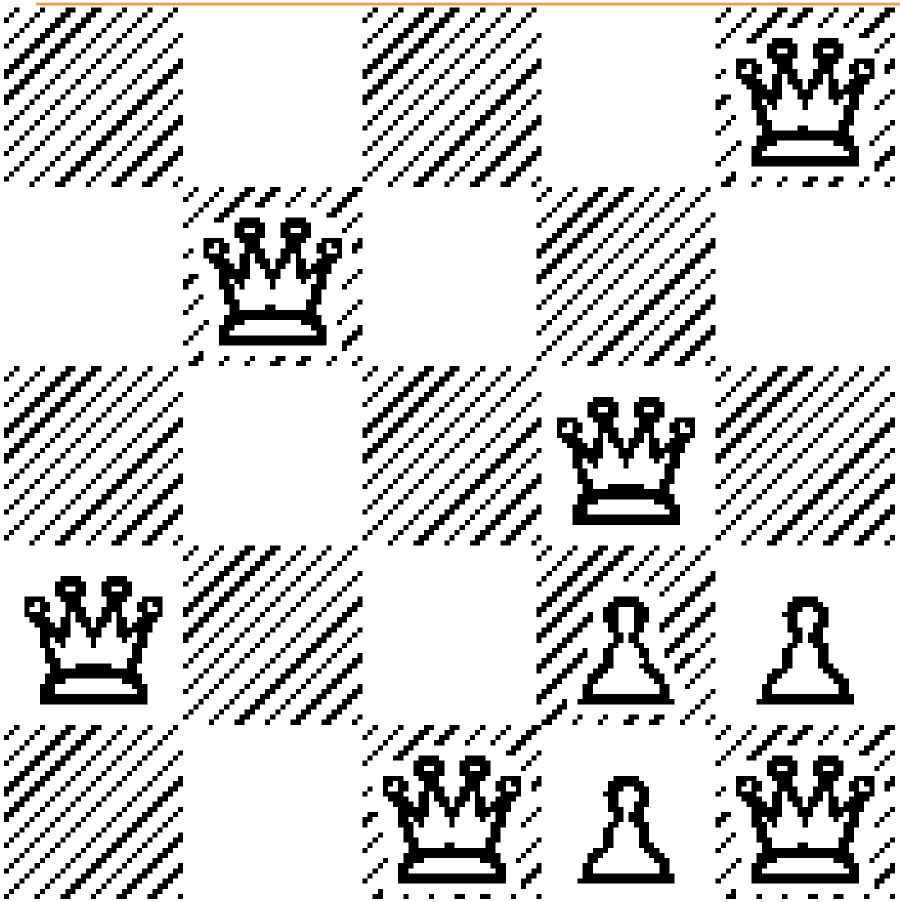
- Let π be a domination-related parameter.
- $s_Q(\pi,S,n)$ is the minimum number of Pawns we need to place on an n-by-n chessboard so that the Queens graph on the remaining squares has π in S.
- Similar definition for other pieces.

Independence-separation: Adding one Queen

- $s_Q(\beta,\{4\},3) = 5$
- $s_Q(\beta,\{5\},4)$ does not exist, since when we put 5 queens on a 4 x 4 board, at least two queens will be on adjacent squares.



Adding one Queen (p. 2)

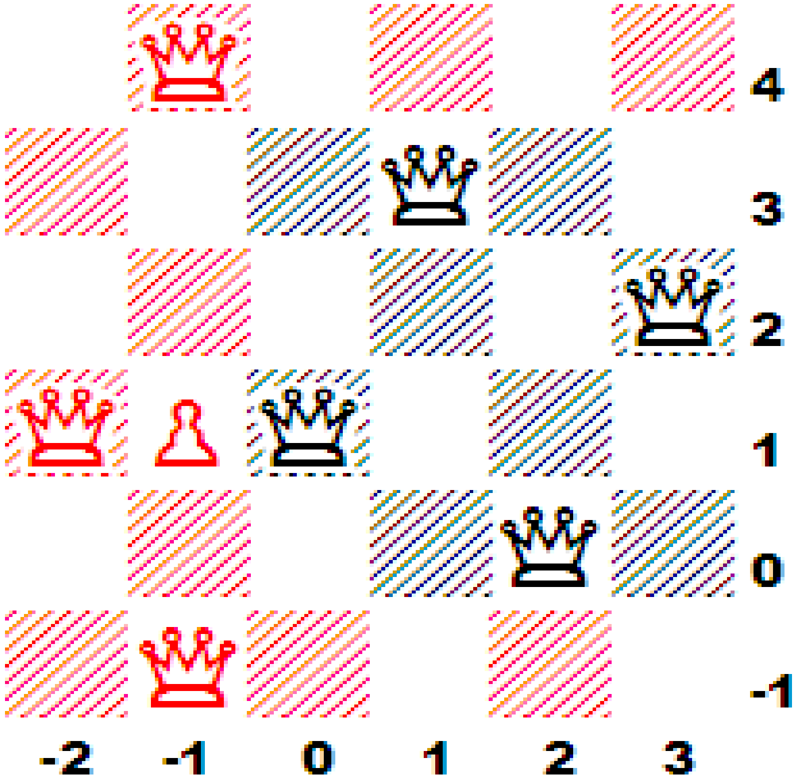


- $s_Q(\beta,\{6\},5) = 3$
  - K. Zhao (1998)
- **Theorem 1**:

  For n>5, $s_Q(\beta,\{n+1\},n) = 1$.

Sketch of Proof of Theorem 1

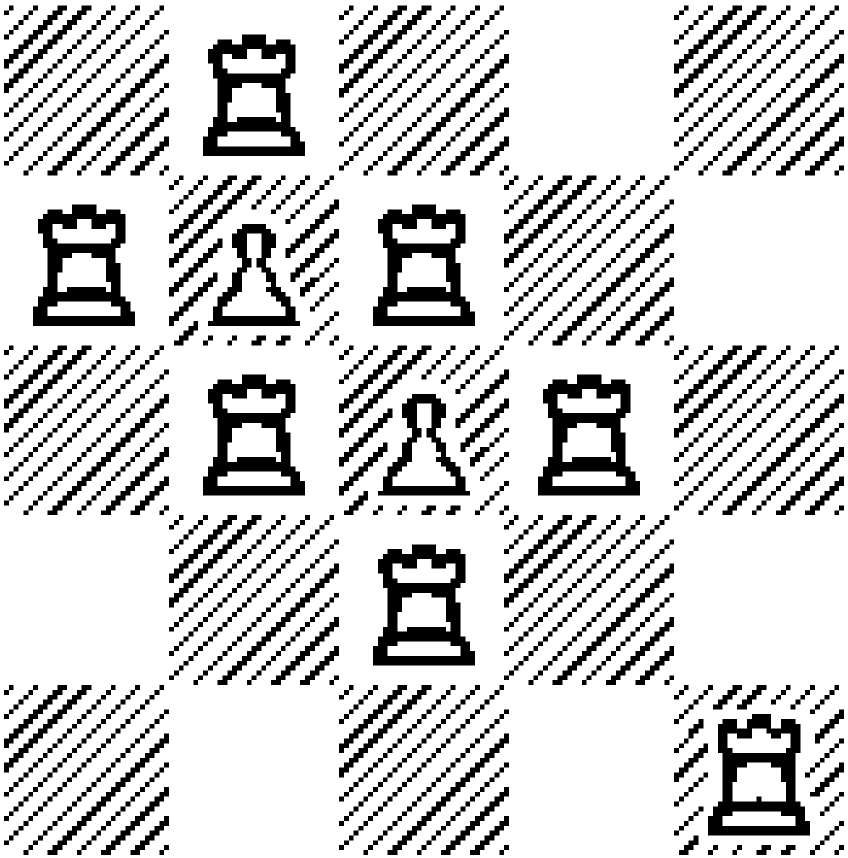
- We take known solutions to the n-queens problem and add extra rows, columns, queens, and a pawn.



Independence-separation: Adding k Queens

- **Theorem 2**: For each k, for large enough N we have $s_Q(\beta,\{N+k\},N) = k$.
  - Proof is like k=1 proof, but uses more patterns.
- For k=2, N=7 is large enough.
- For k=3, N=8 is large enough.
- For $k \geq 4$, $N >$ **25k** is large enough (but how low can we go?)

Rooks independence-separation

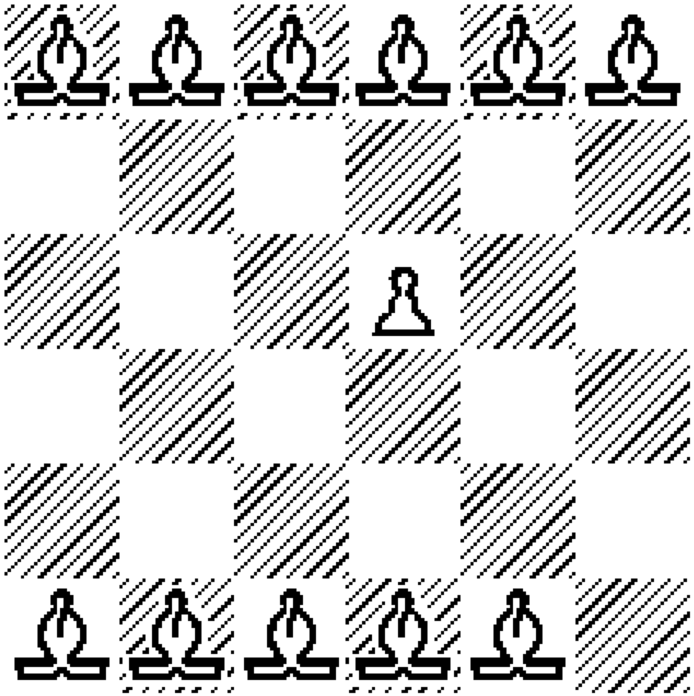


For $N \geq k+2$,

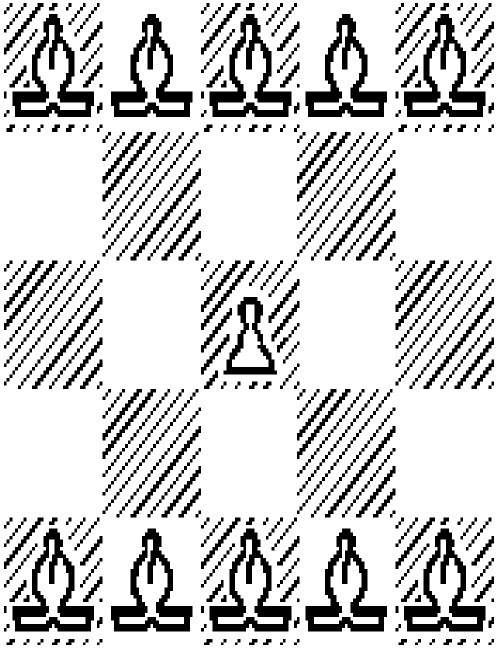
$s_R(\beta,\{N+k\}, N) = k.$

Bishops independence-separation

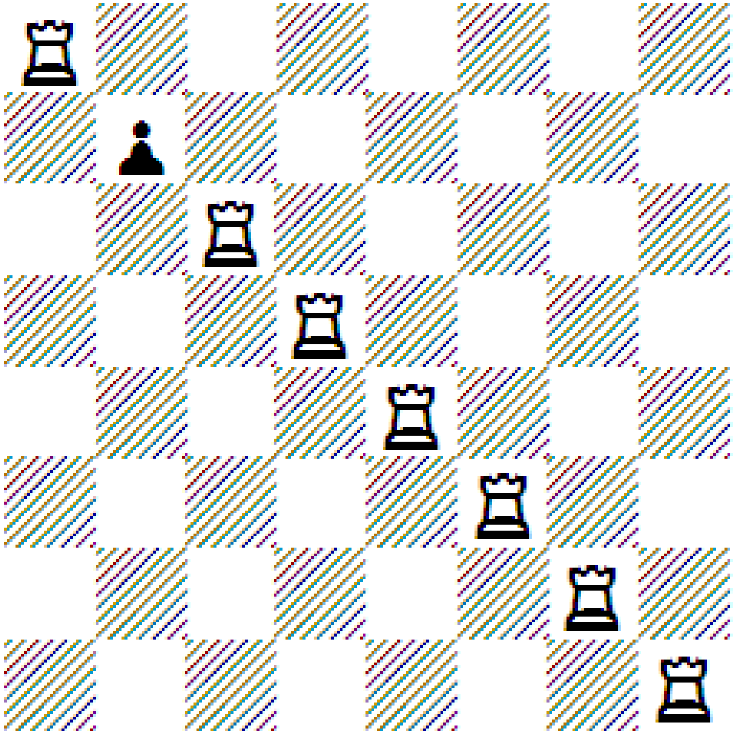
$s_B(\beta,\{2n-1\},n) = 1$ for $n>2$



$s_B(\beta,\{2n\},n) = 1$ for $n>2$ odd

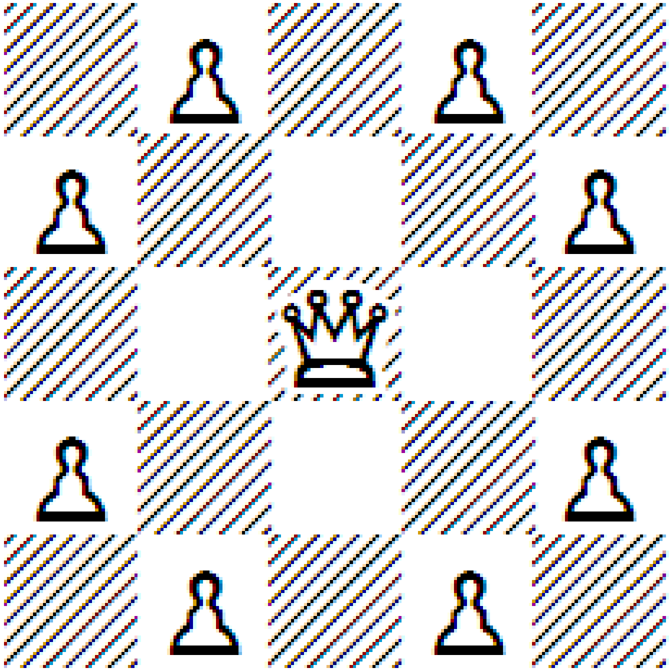


Domination-separation



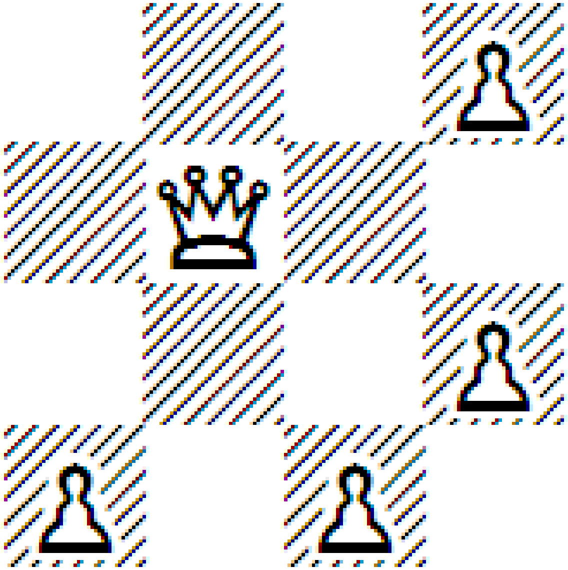
$s_R(\gamma, \{n-1\}, n) = 1$

Domination-separation (p. 2)



For n > 2 odd,

$s_Q(\gamma, \{1\}, n) = n^2 - 4n + 3$

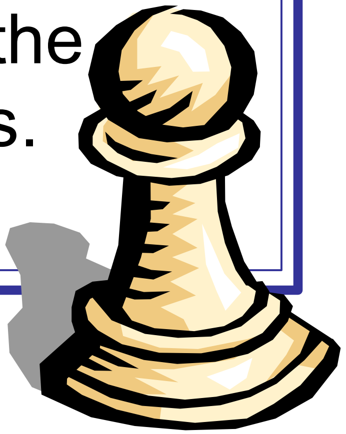


For n > 3 even,

$s_Q(\gamma, \{1\}, n) = n^2 - 4n + 4$

Open Problems

- Alternate boards (rectangular, toroidal, etc.)
- How many solutions?
- Where can the Pawns go?
  - **Proposition**: If N + k mutually nonattacking Queens and k Pawns are placed on an N-by-N board, then none of the Pawns are in the first or last row or column, nor are any Pawns in the squares diagonally adjacent to the corners.



Open Problems (p. 2)

- Alternate domination parameters.
- Alternate pieces (such as Amazon = Q+N)
- Consider "upper π-separation numbers," where we look for the **maximum** number of Pawns needed to put π in the desired set S.

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