



Separating Queens on the Chessboard

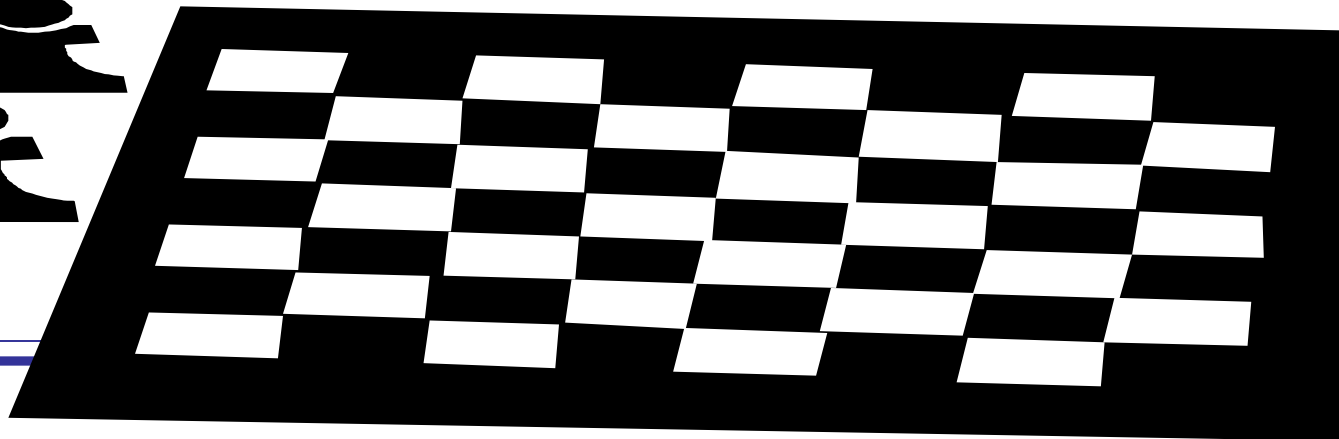
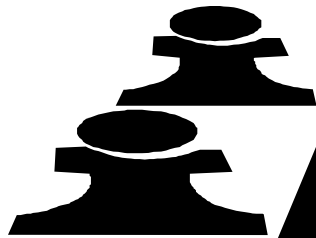
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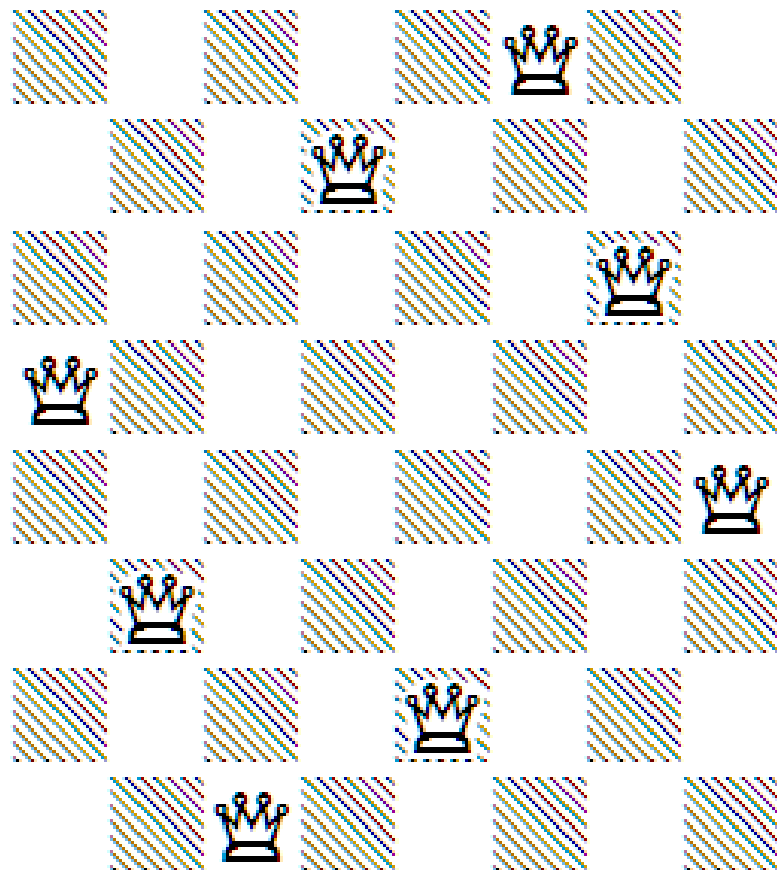
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Acknowledgments

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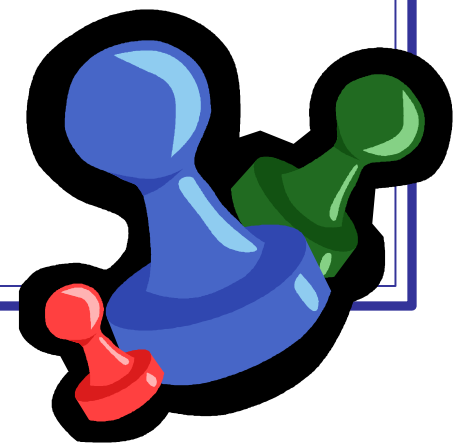
The Eight Queens Problem



- Place eight queens on a standard chessboard so that no two attack each other.
- First posed in 1848.
- Generalized to N queens on an $N \times N$ board.
- Independence number

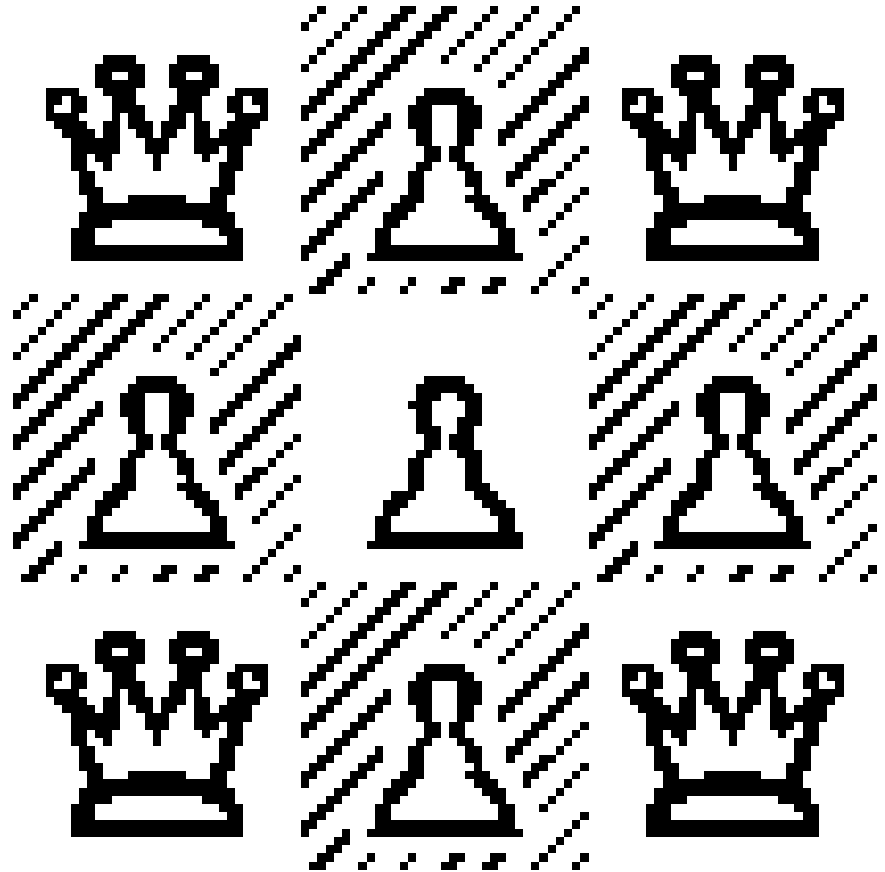
Independence Separation Numbers

- $s_Q(\beta, k, n)$ is the minimum number of Pawns we need to place on an n -by- n chessboard so that the Queens graph on the remaining squares has **independence** number k .
- Similar definition for other pieces.

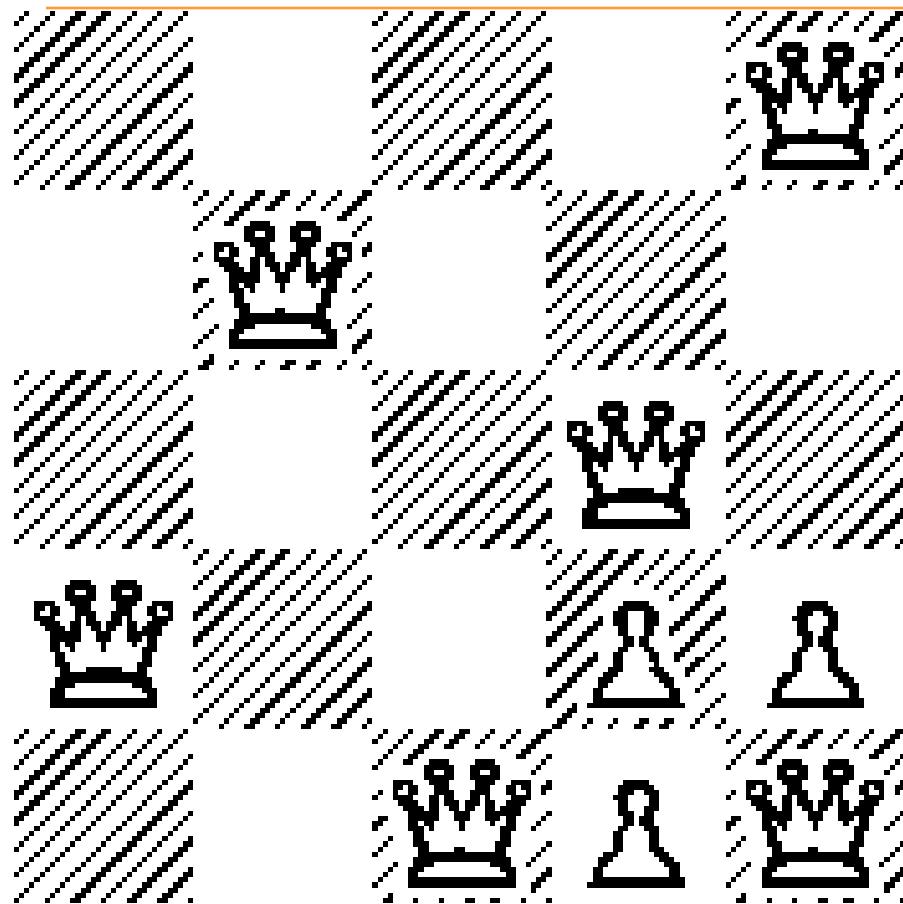


Independence Separation: Adding One Queen

- $s_Q(\beta, 4, 3) = 5$
- $s_Q(\beta, 5, 4)$ does not exist, since when we put 5 queens on a 4 x 4 board, at least two queens will be on adjacent squares.



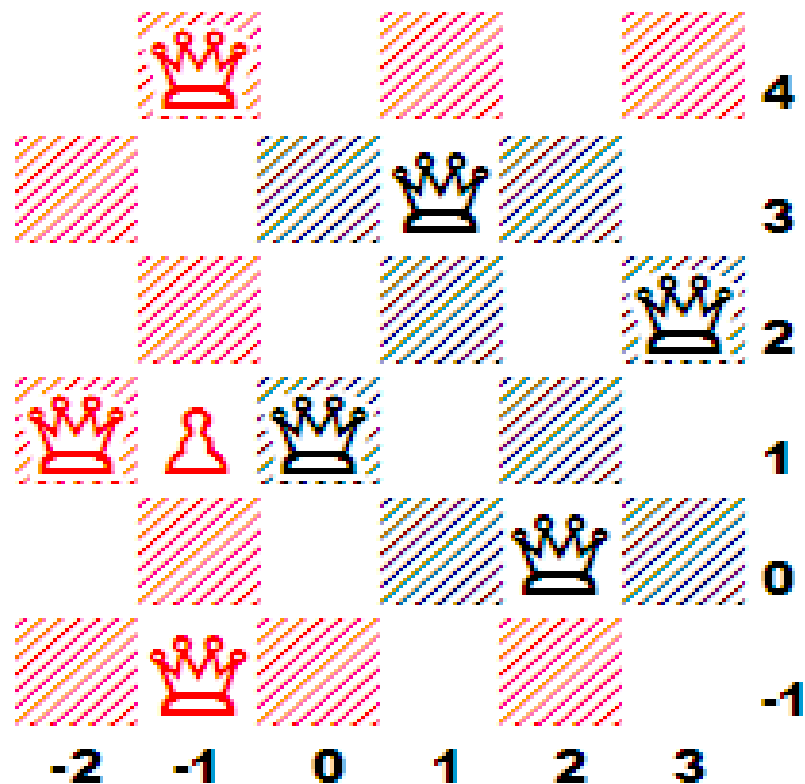
Adding One Queen (p. 2)



- $s_Q(\beta, 6, 5) = 3$
 - K. Zhao (1998)
- **Theorem 1:**
For $n > 5$, $s_Q(\beta, n+1, n) = 1$.

Sketch of Proof of Theorem 1

- We take known solutions to the n -queens problem and add extra rows, columns, queens, and a pawn.

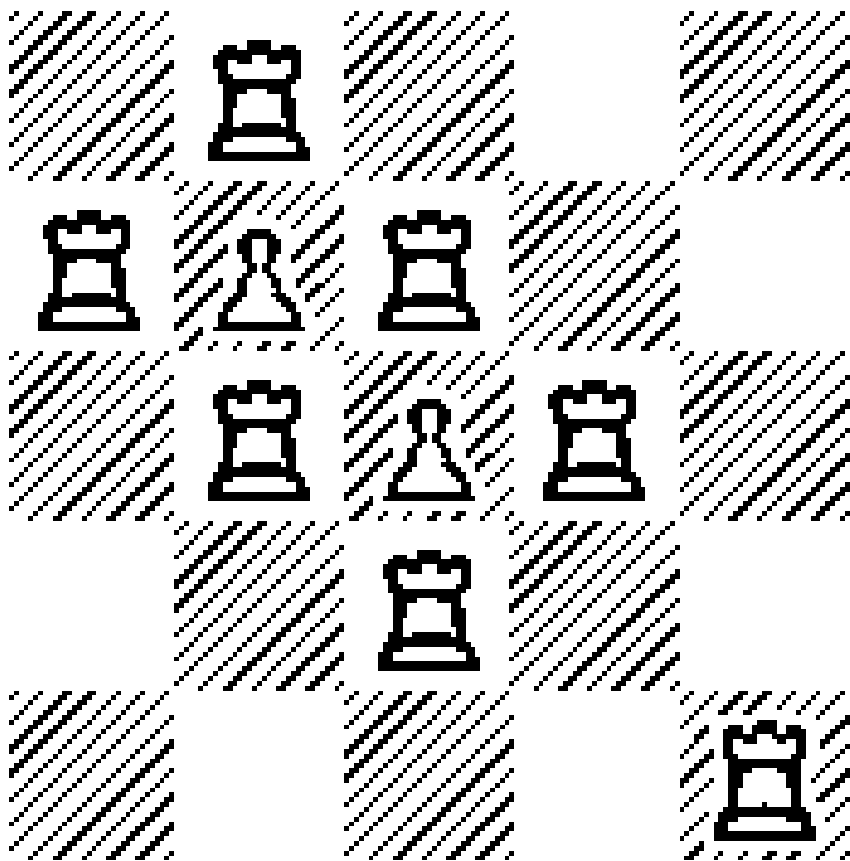


Independence Separation: Adding k Queens

- **Theorem 2:** For each k , for large enough N we have $s_Q(\beta, N+k, N) = k$.
 - Proof is like $k=1$ proof, but uses more patterns.
- For $k=2$, $N=7$ is large enough.
- For $k=3$, $N=8$ is large enough.
- For $k \geq 4$, $N > \mathbf{25k}$ is large enough (but how low can we go?)

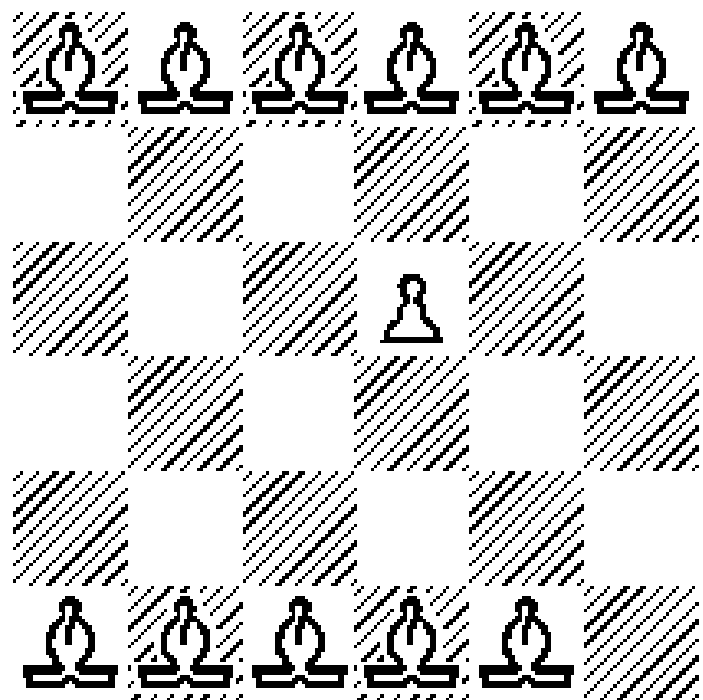
Rooks Independence Separation

For $N \geq k+2$,
 $s_R(\beta, N+k, N) = k.$

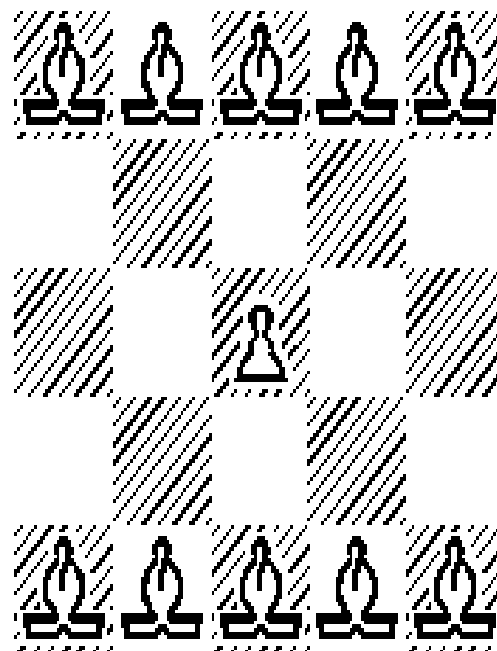


Bishops Independence Separation

$$s_B(\beta, 2n-1, n) = 1 \text{ for } n > 2$$

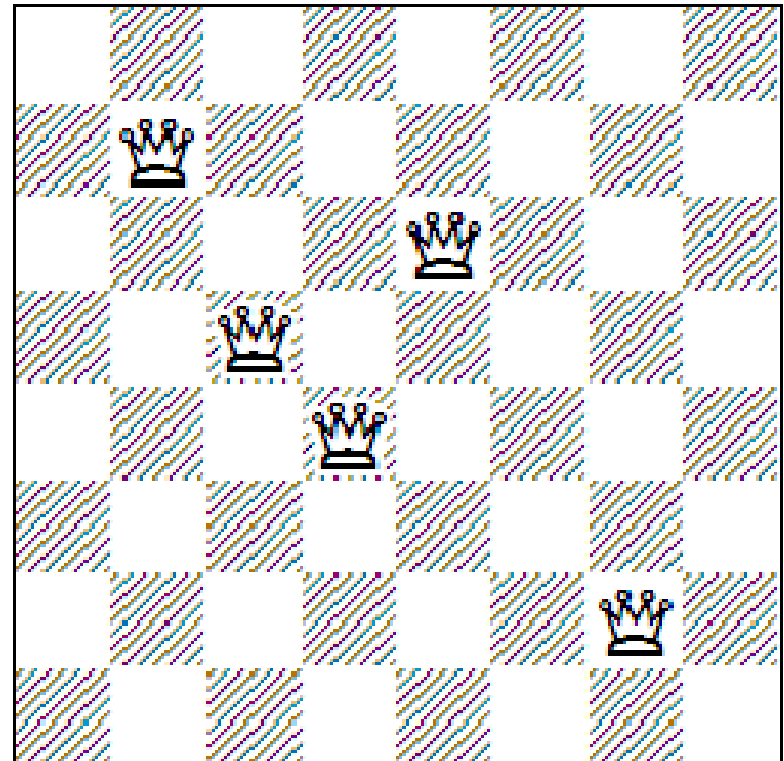


$$s_B(\beta, 2n, n) = 1 \text{ for } n > 2 \text{ odd}$$



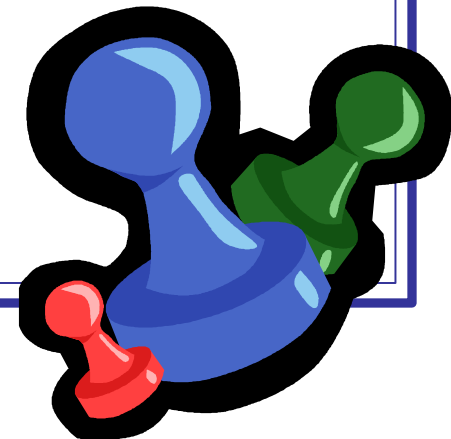
The Five Queens Problem

- Place 5 queens on the chessboard so all squares are either occupied or attacked.
- Introduced in 1862.
- Domination number

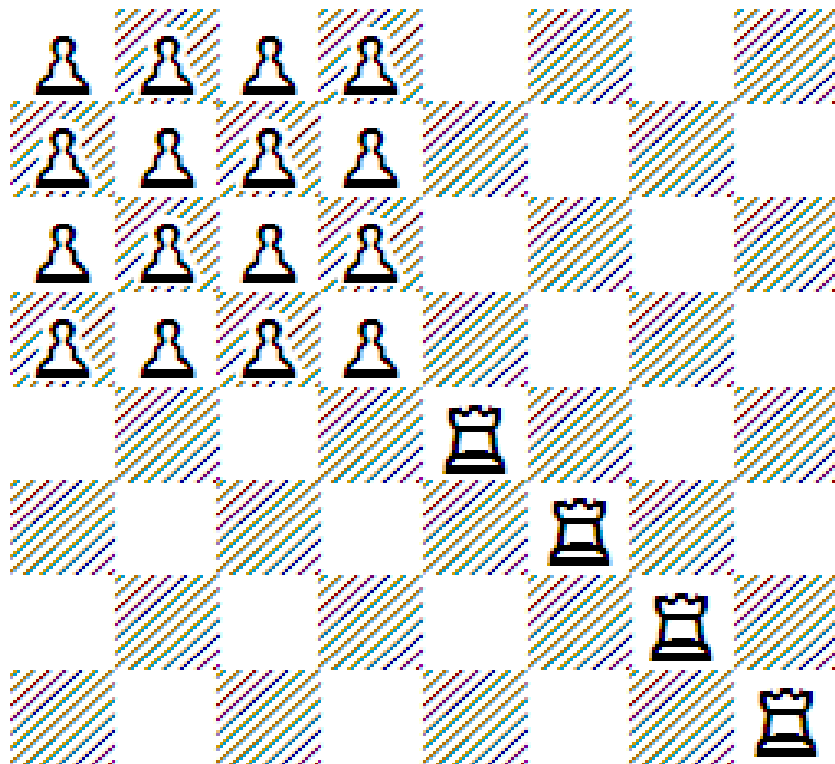


Domination Separation Numbers

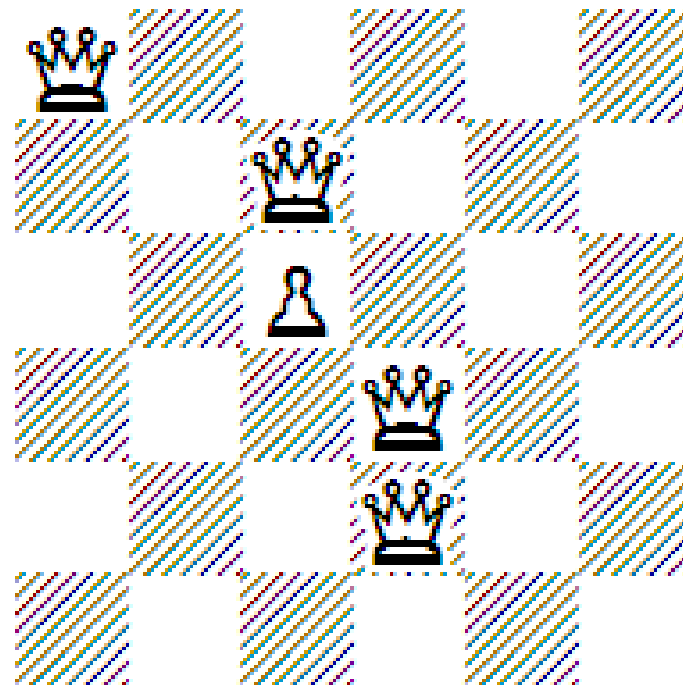
- $s_Q(\gamma, k, n)$ is the minimum number of Pawns we need to place on an n -by- n chessboard so that the Queens graph on the remaining squares has **domination** number k .
- Similar definition for other pieces.



Domination Separation

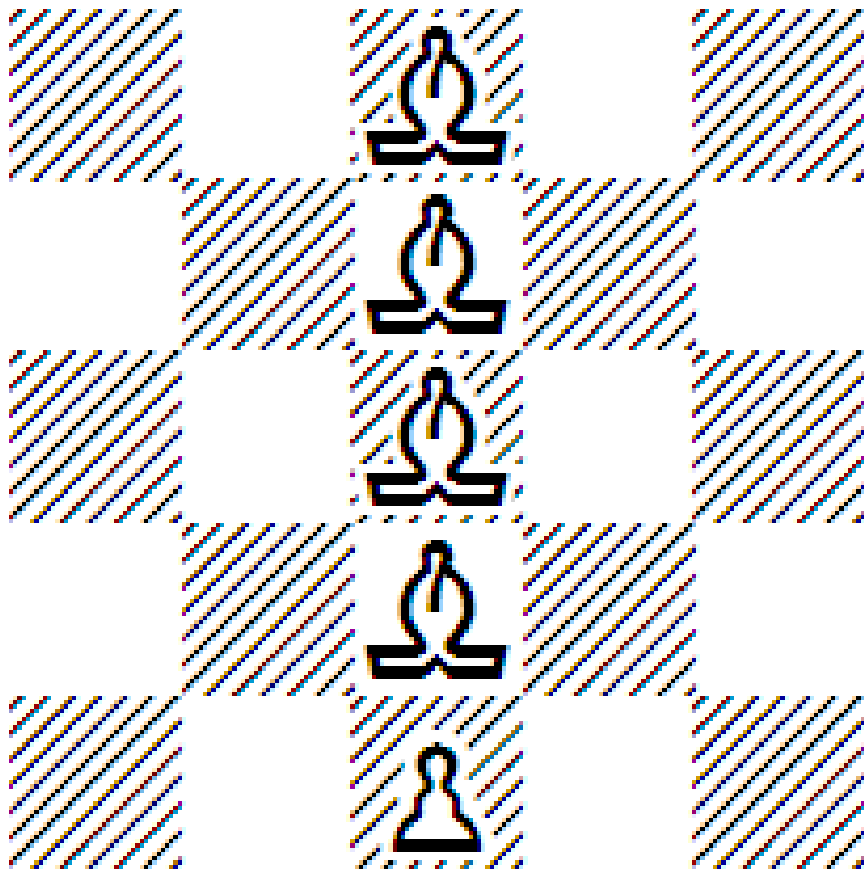


$$s_R(\gamma, n-k, n) = k^2 \text{ for } n \geq k$$



$$s_Q(\gamma, 4, 6) = 1$$

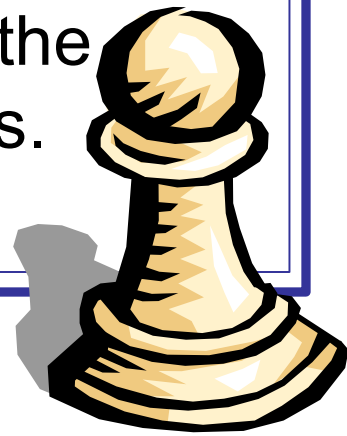
Domination Separation (p. 2)



- For $n > 2$ odd,
 $s_B(\gamma, n-1, n) = 1$
- For $n \geq 2$ even,
 $s_B(\gamma, n-1, n) > 1$

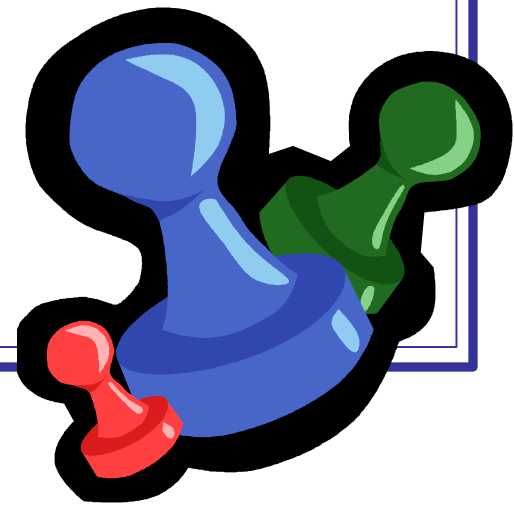
Open Problems

- Alternate boards (rectangular, toroidal, etc.)
- How many solutions?
- Where can the Pawns go?
 - **Proposition:** If $N + k$ mutually nonattacking Queens and k Pawns are placed on an N -by- N board, then none of the Pawns are in the first or last row or column, nor are any Pawns in the squares diagonally adjacent to the corners.



Open Problems (p. 2)

- Alternate domination parameters.
- Alternate pieces (such as Amazon = Q+N)
- Consider “upper π separation numbers,” where we look for the **maximum** number of Pawns needed to get a particular value for π .



References

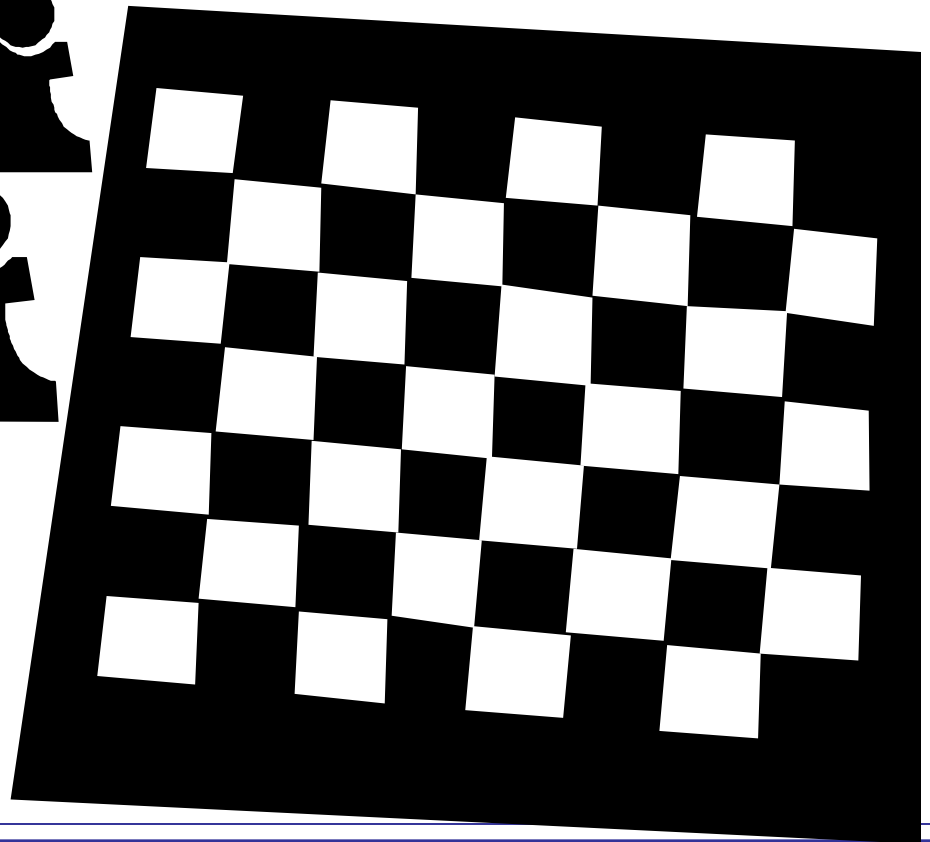
- Chatham, Fricke, and Skaggs, The Queens Separation Problem, *Utilitas Mathematica* 69 (2006), 129-141.
 - Preprint at <http://people.moreheadstate.edu/fs/d.chatham/queenssep.pdf>
- The N+k Queens Problem Page
 - <http://people.moreheadstate.edu/fs/d.chatham/n+kqueens.html>

References (p. 2)

- Watkins, John J. (2004). *Across the Board: The Mathematics of Chess Problems*. Princeton: Princeton University Press. ISBN 0-691-11503-6.
- Zhao, Kaiyan (1998), *The Combinatorics of Chessboards* (Ph. D. Thesis), CUNY.

Your move!

- Any questions?



Counting $N + 1$ Queens Solutions

N	Number of solutions	N	Number of solutions
4	0	11	11,152
5	0	12	65,172
6	16	13	437,848
7	20	14	3,118,664
8	128	15	23,387,448
9	396	16	183,463,680
10	2,288	17	1,474,699,536

Fundamental Solutions

N	Number of solutions	N	Number of solutions
4	0	11	1,403
5	0	12	8,214
6	2	13	54,756
7	3	14	389,833
8	16	15	2,923,757
9	52	16	22,932,960
10	286		

Counting $N + 2$ Queens Solutions

N	Number of solutions	N	Number of solutions
5	0	10	1,304
6	0	11	12,452
7	4	12	105,012
8	44		
9	280		

Fundamental Solutions

N	Number of solutions	N	Number of solutions
5	0	10	164
6	0	11	1,572
7	1	12	13,133
8	6		
9	37		

Counting $N + 3$ Queens Solutions

N	Number of solutions
7	0
8	8
9	44
10	528
11	5,976

Fundamental Solutions

N	Number of solutions
7	0
8	1
9	6
10	66
11	751