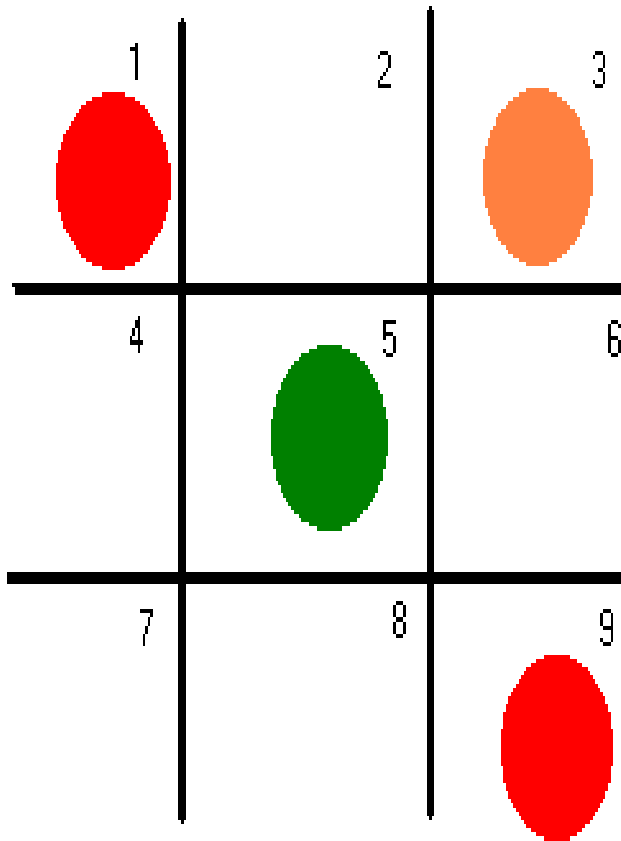


Winning Strategy for Traffic Lights and Beer Square

Doug Chatham

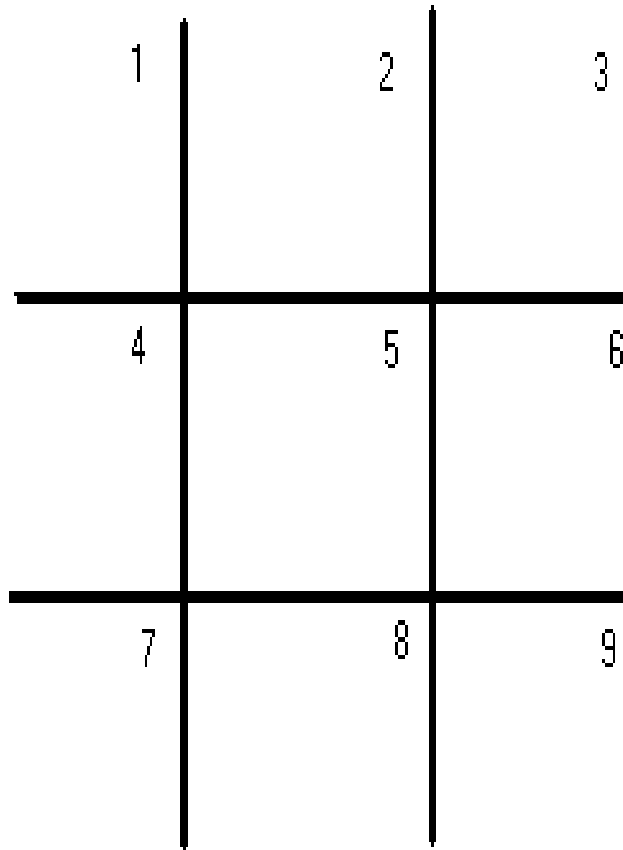
Morehead State University

Traffic Lights



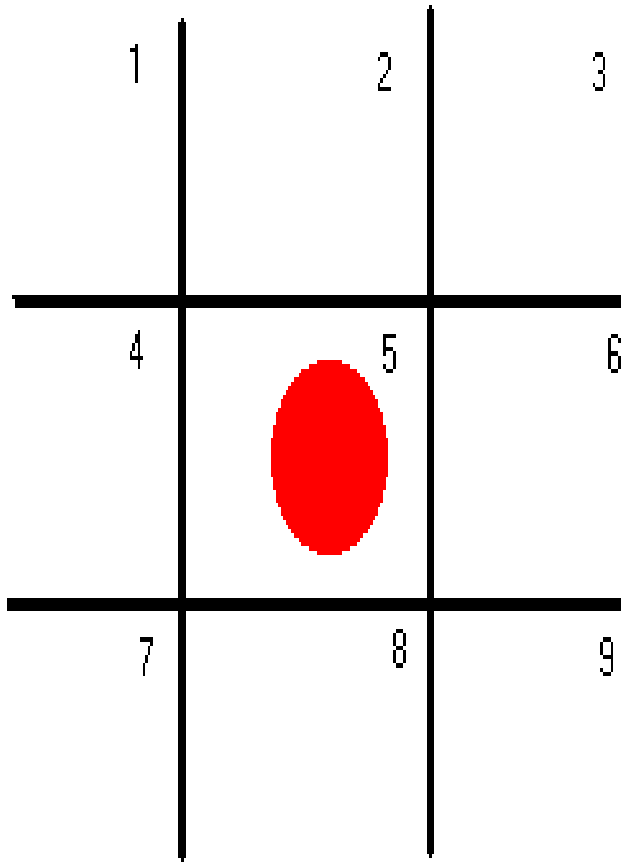
- Tic-tac-toe variant invented by Alan Parr
- <http://www.nrich.maths.org.uk/maths/journal/mar98/game1/>

Traffic Lights rules



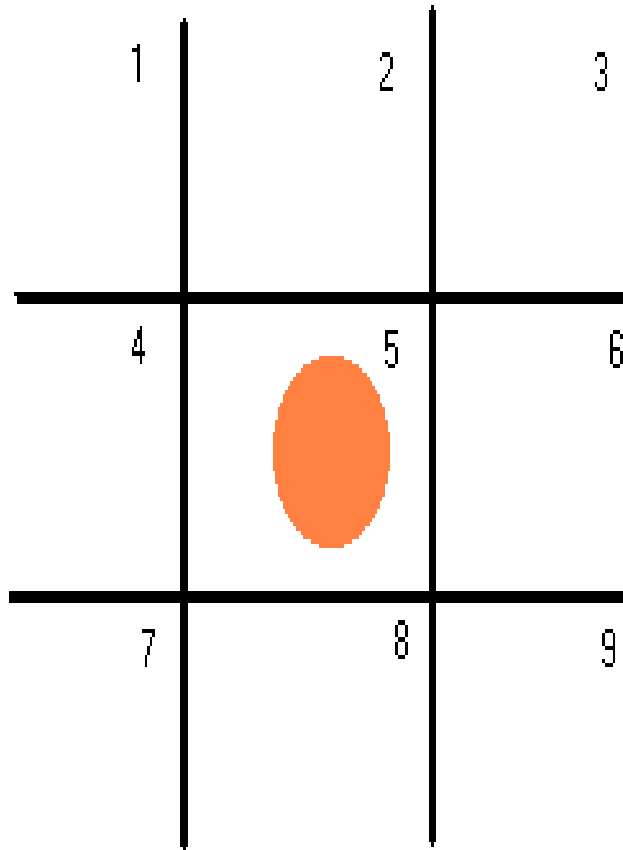
- Play starts on an empty 3 x 3 board
- On his or her turn, a player may do ONE of the following:

Traffic Lights rules



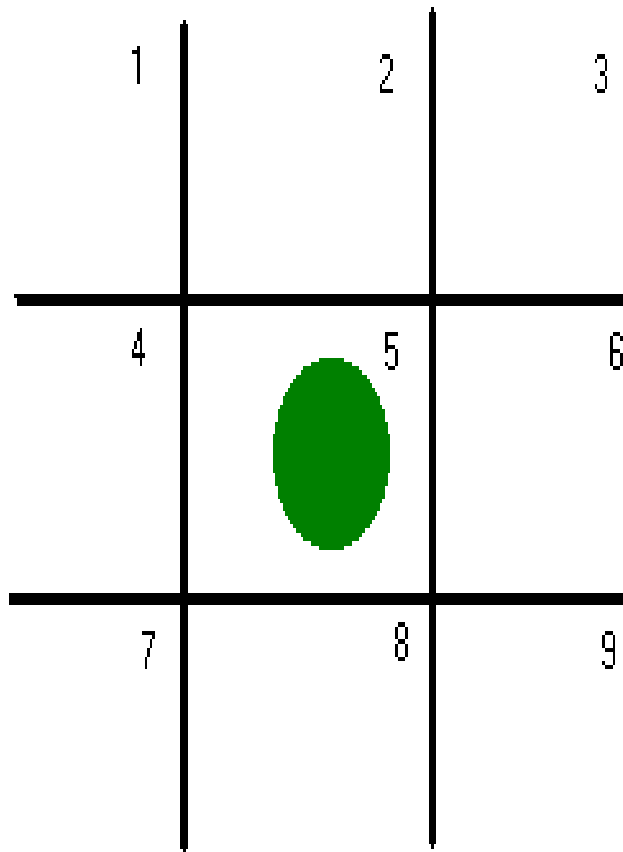
- Place a red counter in any empty square

Traffic Lights rules



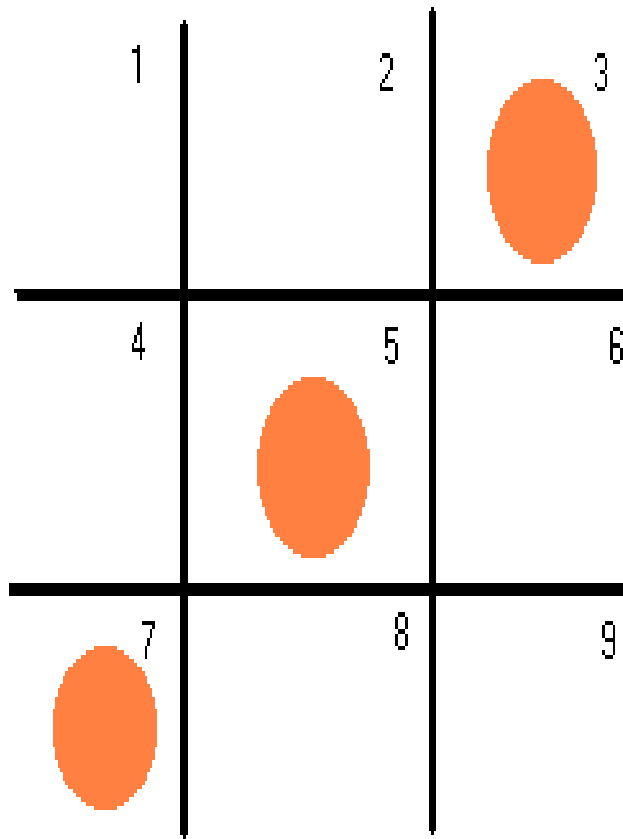
- Place a red counter in any empty square
- OR replace any red counter on the board with a orange one

Traffic Lights rules



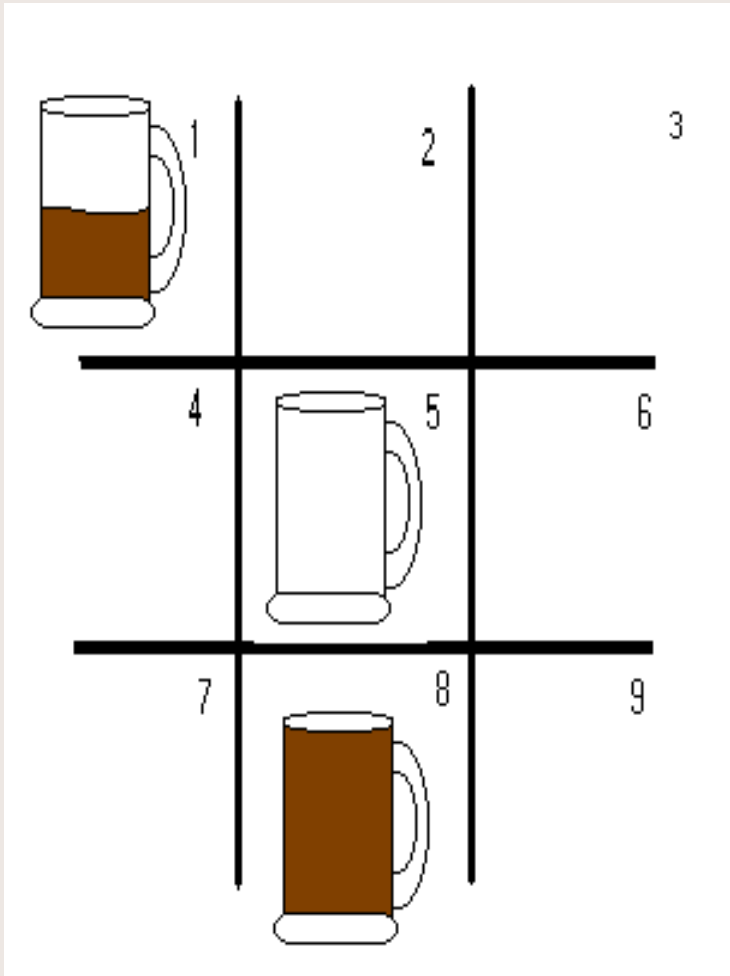
- Place a red counter in any empty square
- OR replace any red counter on the board with a orange one
- OR replace an orange counter on the board with a green one

Traffic Lights rules



- A player wins if he or she causes a row (horizontal, vertical, or diagonal) of 3 counters to have the same color.

Beer Square

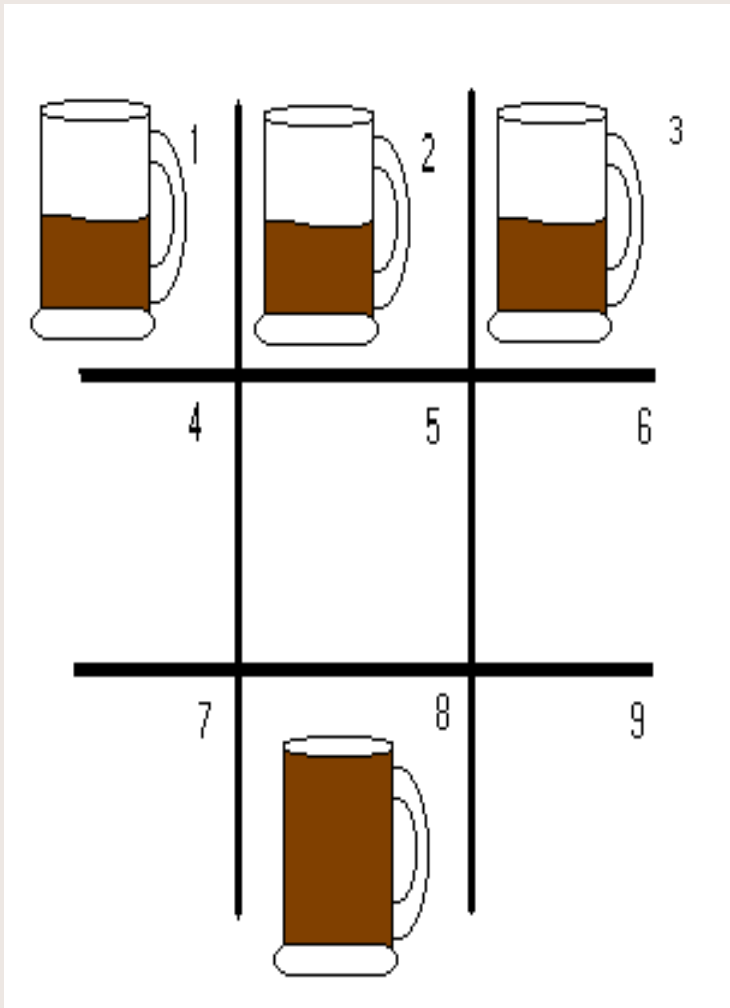


- Invented by Dan Glimne
- <http://www.bisforbeer.com/fun/beersquares.html>

Beer Square rules

- On his or her turn a player may
 - Place a mug on an empty mat in either a full, half-full, or empty state, OR
 - Take a full mug from a mat and put it back in a half-full or empty state, OR
 - Take a half-empty mug from a mat and put it back in an empty state.

Beer Square rules

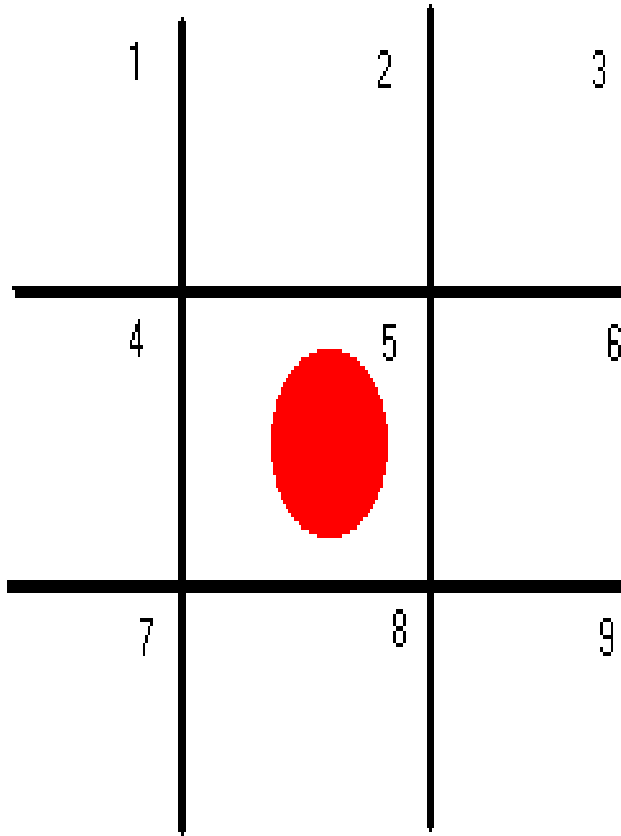


- A player wins if he or she causes a row (vertical, horizontal, or diagonal) of three mugs to be in the same state.

Traffic Lights: 1st Player Winning Strategy

- Place red counter in center square
- On subsequent turns, if you cannot immediately win, do on square $10-n$ what the 2nd player just did on square n
 - If 2nd player changed the counter in square 5, change it again.

Proof that the strategy works

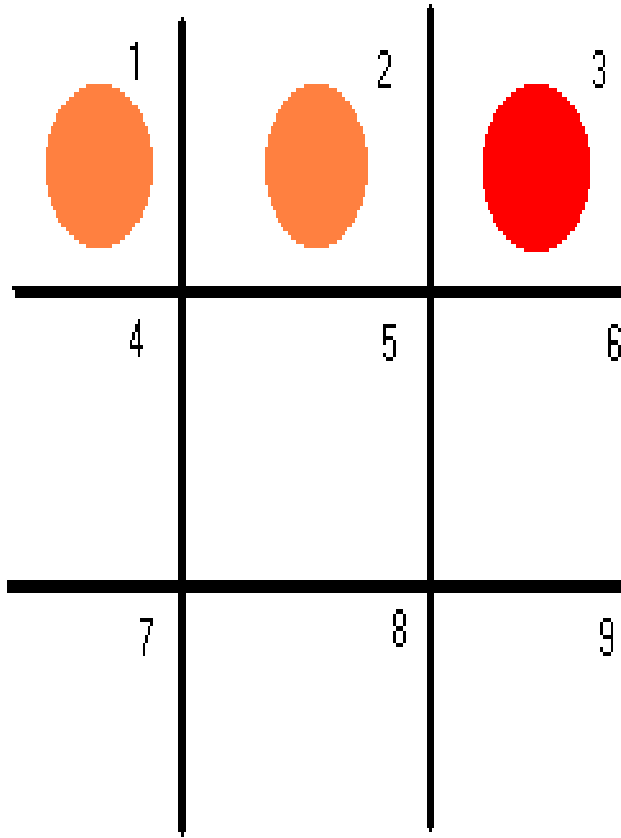


- After you place a red counter in the center, the second player's *only* safe move is to change that counter to orange.
- Your second move changes that orange to green.

Proof that the strategy works

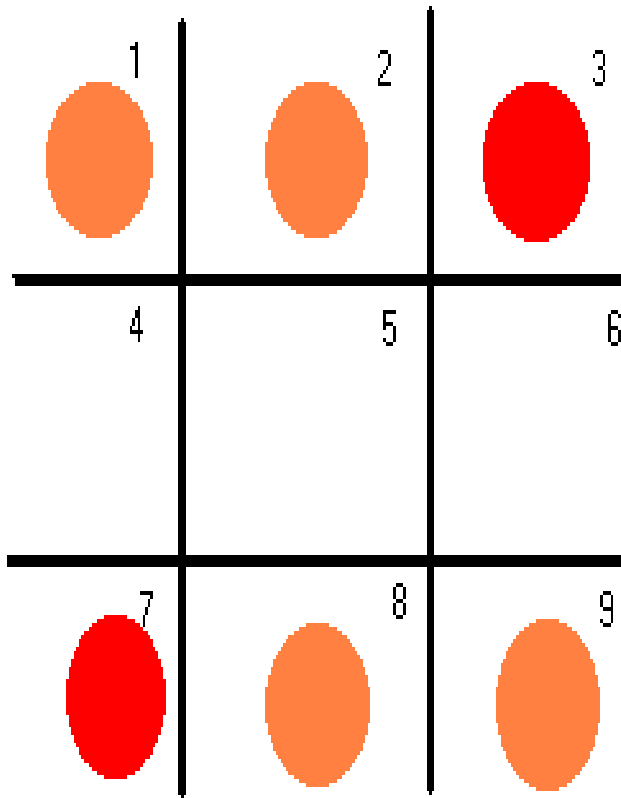
- Suppose it's the second player's turn and he/she has an immediate winning move.
- So the 2nd player can complete a row.
- That row either goes through the center or it doesn't.

Proof that the strategy works



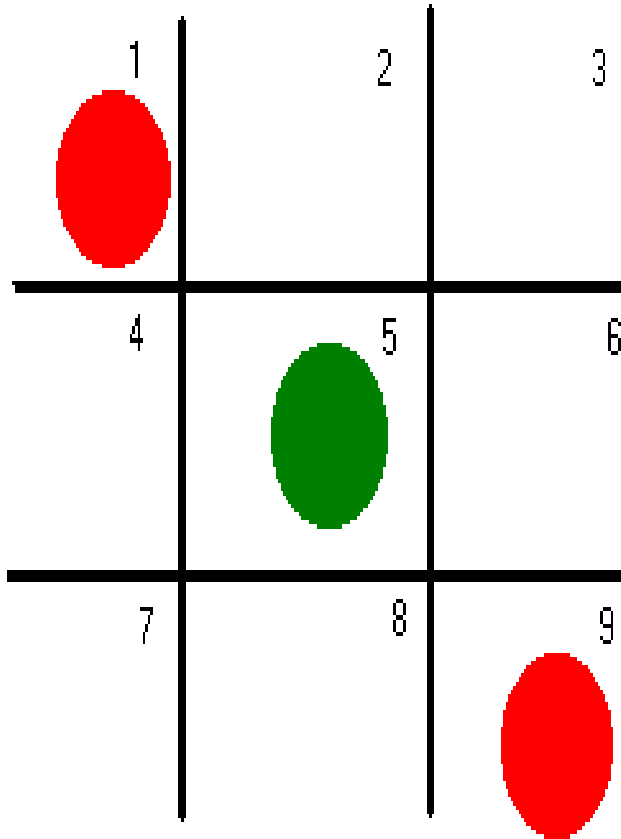
- If the row (I, J, K) does not go through the center, then the row $(10-I, 10-J, 10-K)$ is a distinct row that the 2nd player can complete.

Proof that the strategy works



- If the row (I, J, K) does not go through the center, then the row $(10-I, 10-J, 10-K)$ is a distinct row that the 2nd player can complete.
- **One of these rows must have been there at your last turn!**

Proof that the strategy works



- If the row goes through the center, the two ends must be in the same state.
- It will take more than one move to finish that row.

Beer Square: 1st Player Winning Strategy

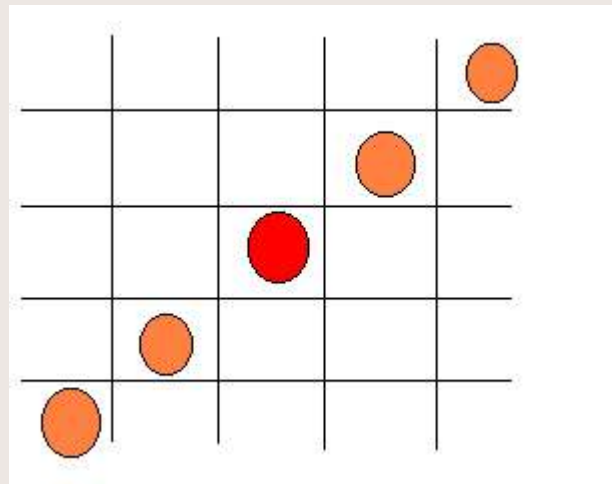
- Place an *empty* mug in the center square.
- On subsequent turns, if you cannot immediately win, do on square $10-n$ what the 2nd player just did on square n .
- The proof that this strategy works is similar to the proof for Traffic Lights.

4x4 variations

- 2nd Player Win
- Whatever 1st Player does on square n , do on square $17-n$, unless there is another move that immediately wins the game.
- Similar strategies also work for $2n \times 2n$ versions of Traffic Lights and Beer Square.

5x5

- Analog of 3x3 strategy fails for 5x5 Traffic Lights, but **not** for 5x5 Beer Square.
- Generalizes to $(2n+1) \times (2n+1)$ versions.



Open Questions

- The other player's strategy?
- Strategy for $(2n+1) \times (2n+1)$ Traffic Lights?
- Shorter winning rows?
- Non-square boards?
- Higher dimensions?
- Losing (avoidance) versions?