No-Deal Solitaire

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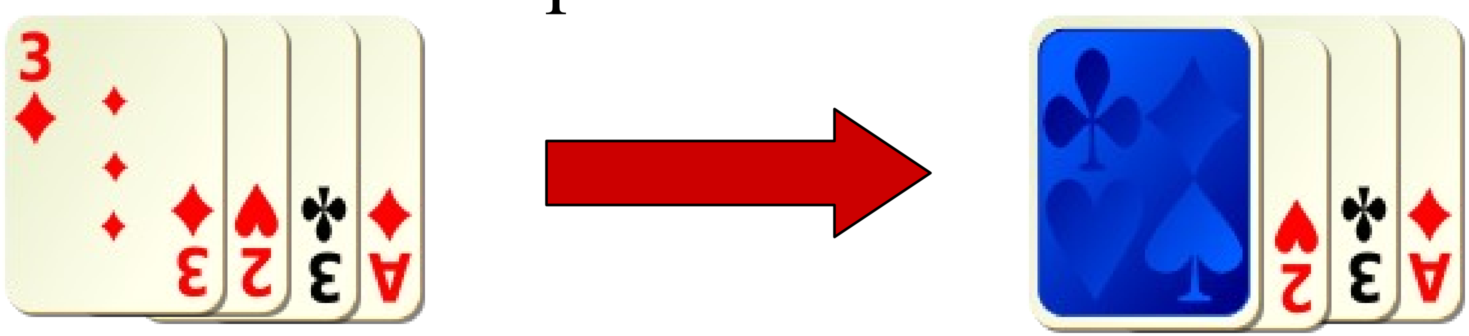
Outline

- Game Rules and Variations
- Basic Questions
  - Is it possible to win?
  - Can winning strategy be found easily?
- Statistics Connections
  - Expected values
  - Effect of different strategies
- Graph Theory Connections
  - Hamiltonian paths and cycles

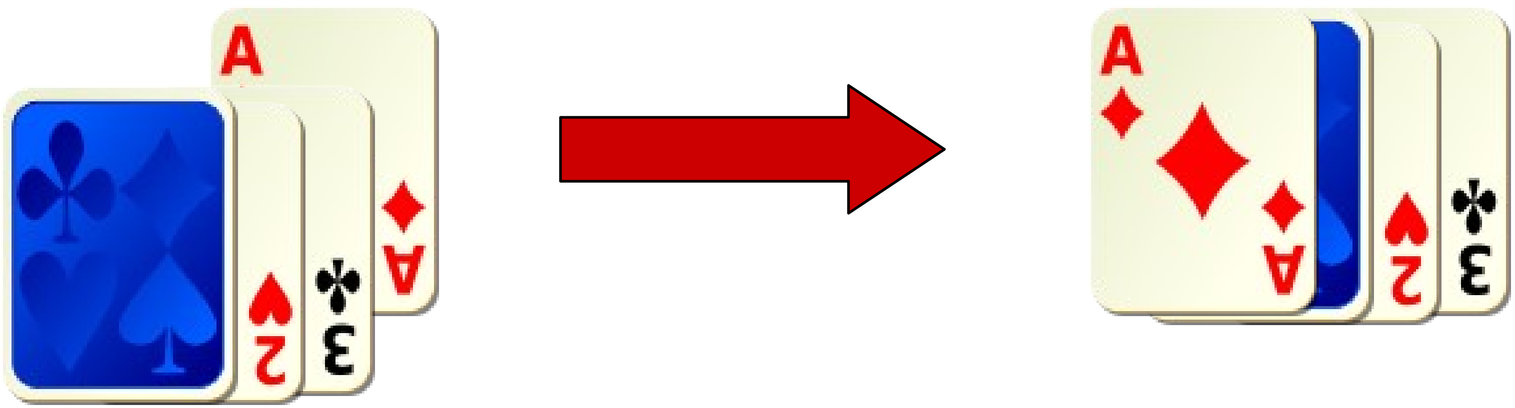
Rules of No-Deal Solitaire

- Equipment: One standard 52-card deck
- Shuffle the deck. Turn the deck over so all the cards are face-up.
- Look at the top card and decide whether to go forward or back.
- Turn over the top card so that it's face down.



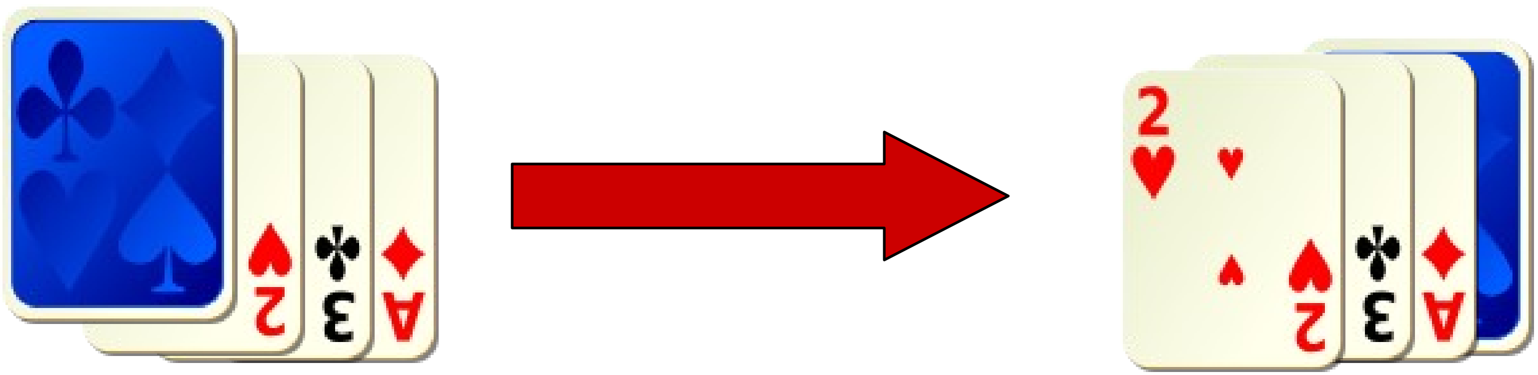
Rules (continued)

- Let k be the number on the top card (with A=1, J=11, Q=12, K=13). [It's 3 here.]
- If you're going forward, take k cards from the top of the deck and move them to the bottom.



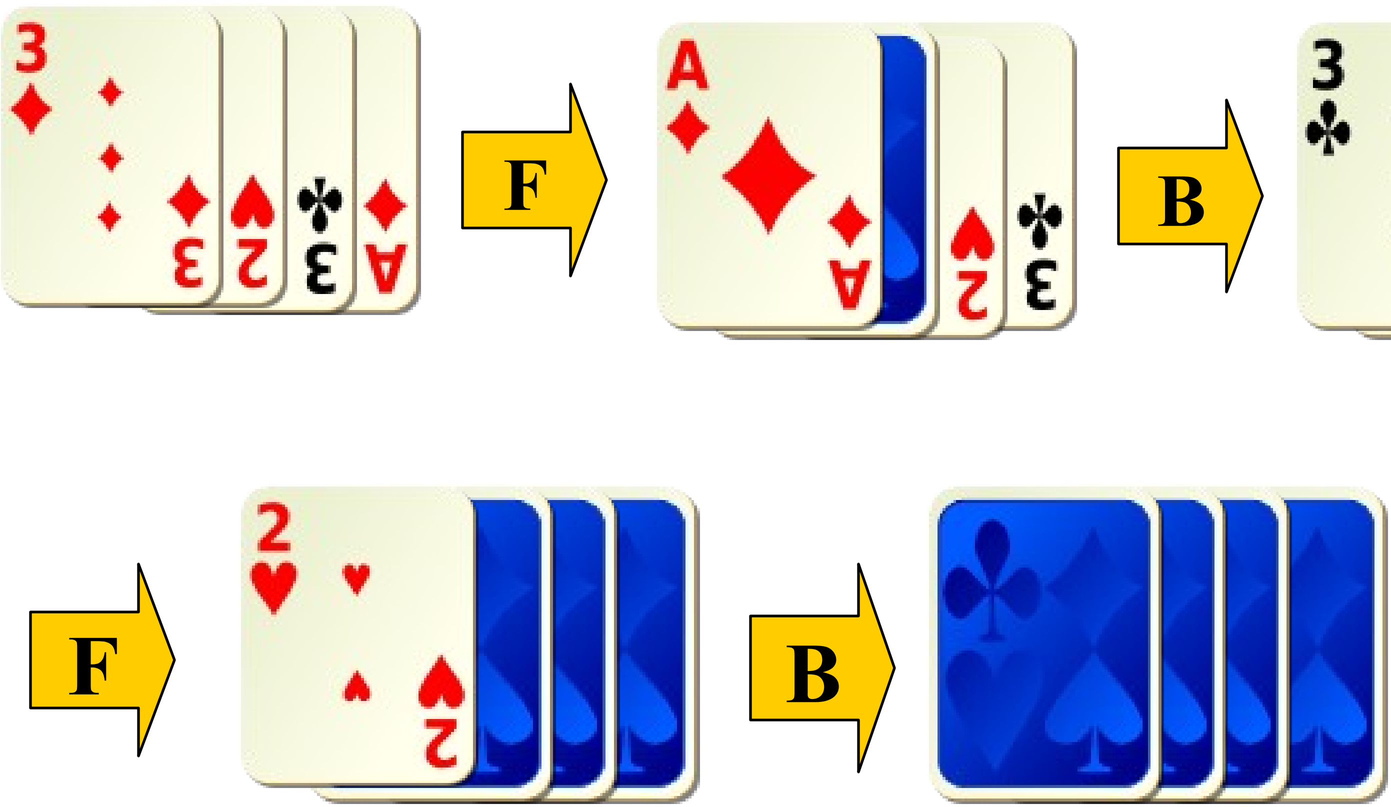
Rules (continued)

- If you're going back, take k cards from the bottom of the deck and move them to the top.



- If the new top card is face up, repeat the last five steps. Otherwise, the game is over.
- If all the cards are face-down, you win.

Demonstration Game



Variations

- Don't play with a full deck. ☺
- Play with Rook deck or numerical Uno cards.
- Play to lose as quickly as possible.
- Declare win if you've turned over most of the cards.
- Keep score (e.g. score = # of overturned cards).
- Allow limited number of repeats.
- If you hit a special card, turn the entire deck over. (Joe Harless)

A deck you can win with

13	11	9	7	5	3	***1***	2	4	6	8	10	12
12	10	8	6	4	2	13	1	3	5	7	9	11
13	11	9	7	5	3	1	2	4	6	8	10	12
12	10	8	6	4	2	13	1	3	5	7	9	11

A deck you can lose quickly with

13	11	9	7	5	3	1	2	4	6	8	10	12
13	10	8	6	4	2	12	1	3	5	7	9	11
13	11	9	7	5	3	1	2	4	6	8	10	12
13	10	8	6	4	2	12	1	3	5	7	9	11

Can we find a winning strategy when one exists?

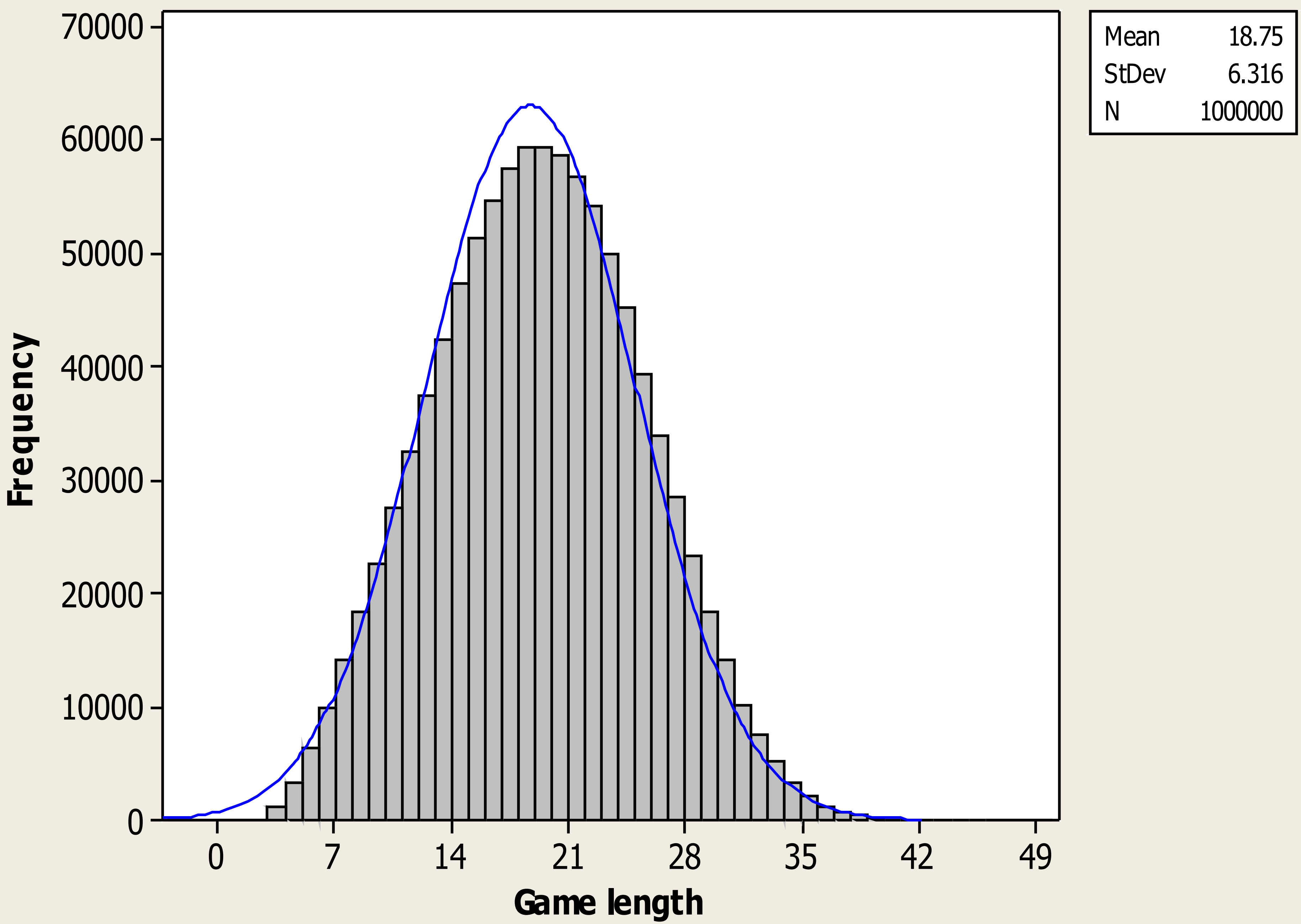
- Yes – it can be done with backtracking.

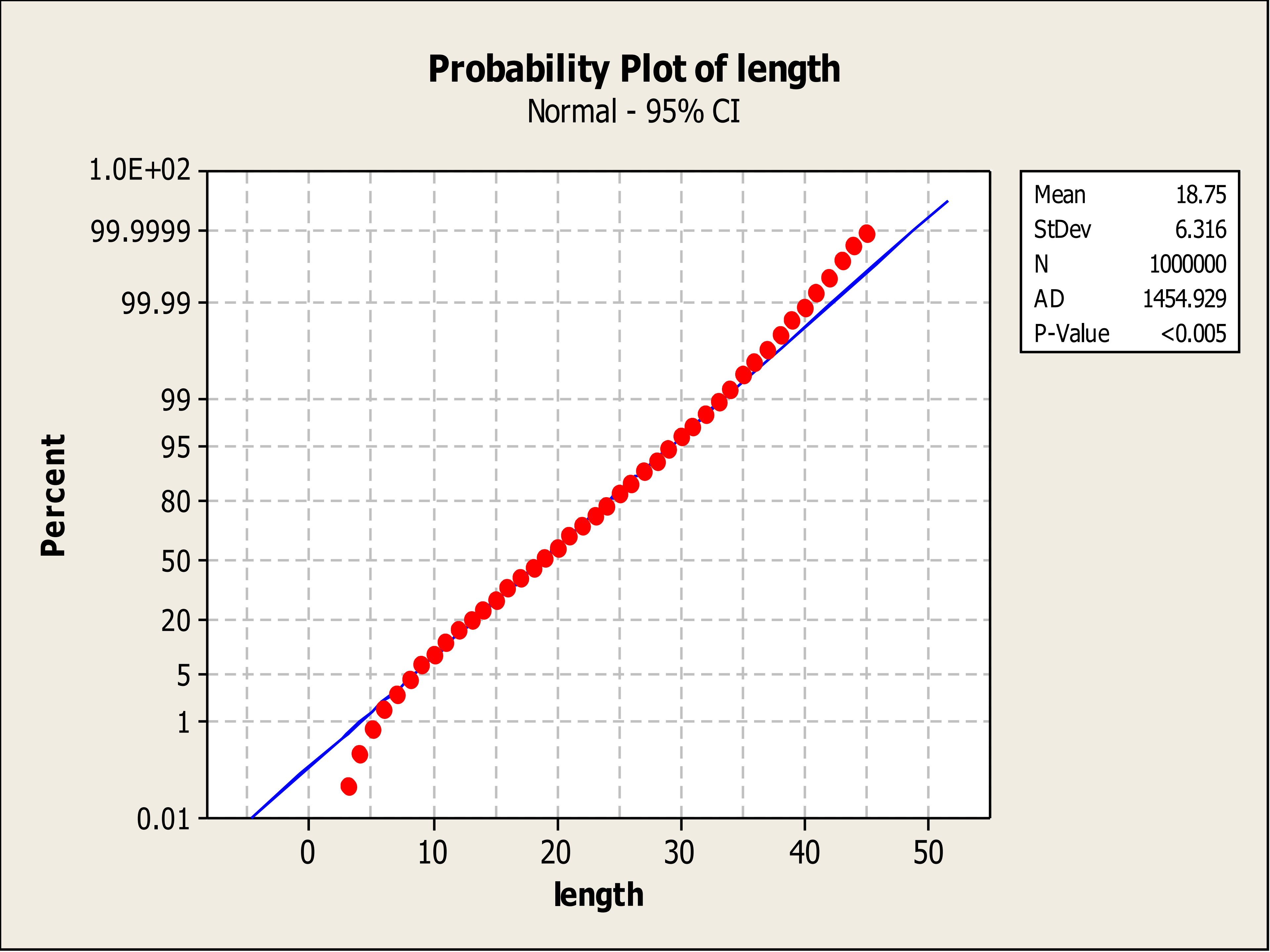
```
procedure getsol(currsol):
    mark currentcard visited
    k = (number on currentcard)
    if (all cards visited):
            print(currsol)
    else:
            if currentcard+k mod decksize not visited:
              getsol(currsol+((currentcard+k) mod decksize))
            if currentcard-k mod decksize not visited:
              getsol(currsol+((currentcard-k) mod decksize))
```

- Can it be done with more clever methods?

Statistics: How long does the average game last?

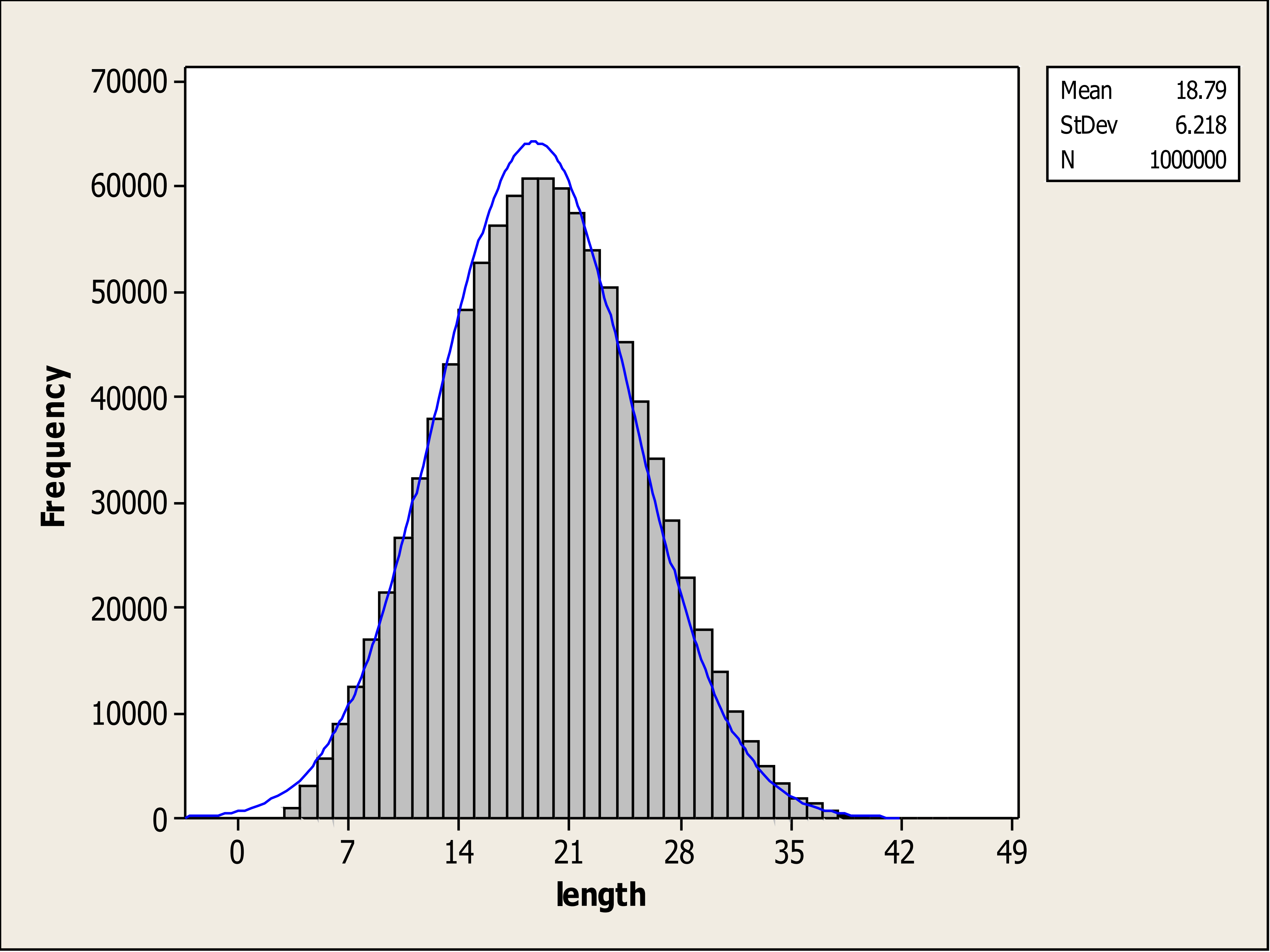
- Assume that player chooses randomly when both options are open.
- Strategy 1: P(player goes forward) = P(player goes back)
  - Python simulation with 1,000,000 decks
  - Expected game length = 18.747456
  - MINITAB histogram on next slide


Mean 18.75
StDev 6.316
N 1000000
Frequency
Game length
0
10000
20000
30000
40000
50000
60000
70000
0
7
14
21
28
35
42
49


Probability Plot of length
Normal - 95% CI
Percent
1.0E+02
99.9999
99.99
99
95
80
50
20
5
1
0.01
0
10
20
30
40
50
length
Mean 18.75
StDev 6.316
N 1000000
AD 1454.929
P-Value <0.005

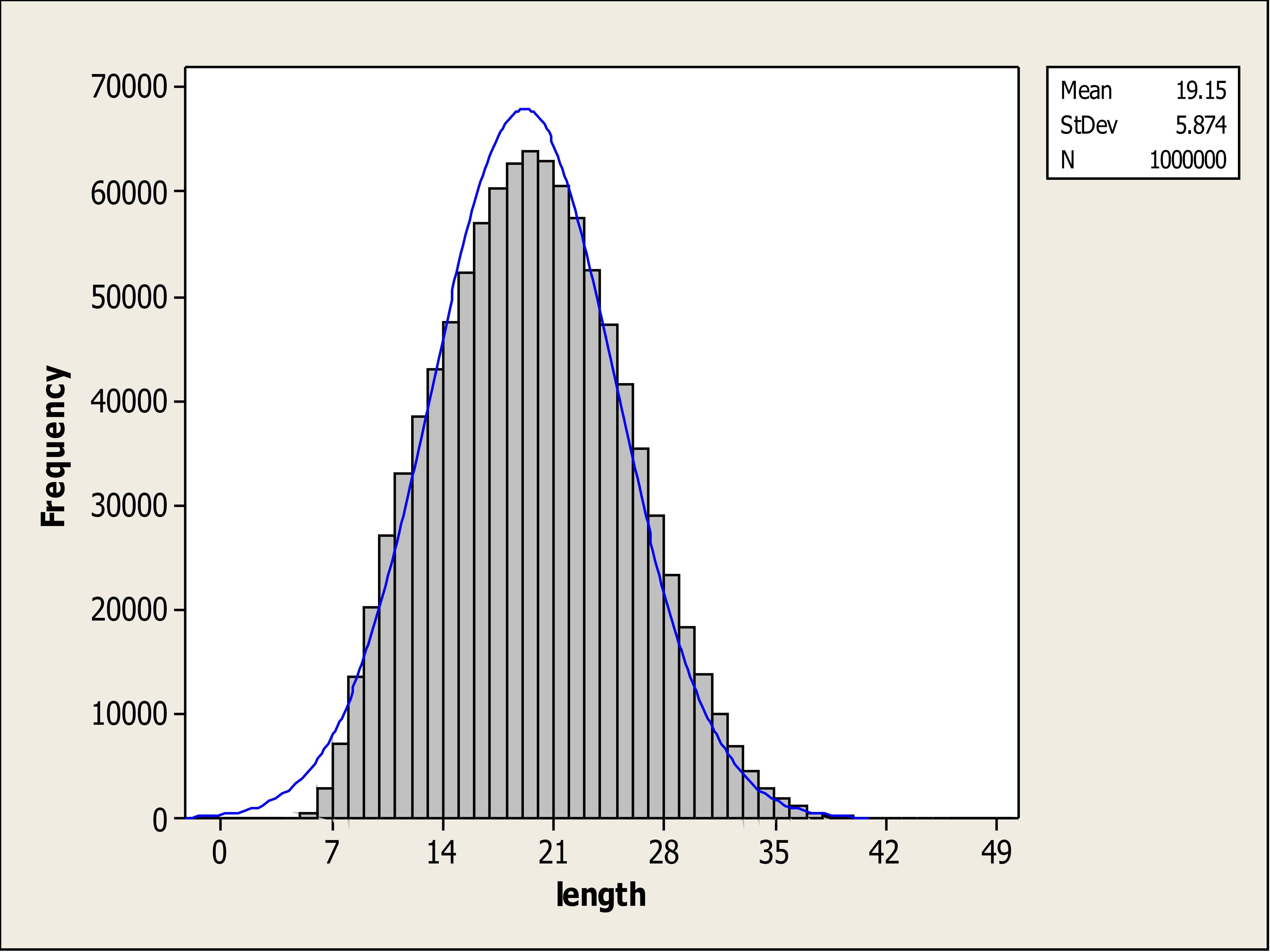
Average game (continued)

- Strategy 2: P(forward) = 2 * P(back)
  - Python simulation of 1,000,000 decks
  - Average game = 18.787904
  - MINITAB histogram on next slide


Mean 18.79
StDev 6.218
N 1000000
Frequency
70000
60000
50000
40000
30000
20000
10000
0
0
7
14
21
28
35
42
49
length

Different strategy

- Strategy 3: Instead of moving randomly, player goes forward if the 13 cards in front are less visited than the 13 cards in other direction.
  - Python simulation of 1,000,000 decks
  - Average game = 19.148144
  - MINITAB histogram on next slide



Mean 19.15
StDev 5.874
N 1000000
Frequency
70000
60000
50000
40000
30000
20000
10000
0
0
7
14
21
28
35
42
49
length

Graph theory: Hamiltonian paths and cycles

- Deck can be treated as a directed graph
  - Each card is a vertex.
  - Directed edge from a to b if player can move in one step from a to b.
- Hamiltonian path: path that visits each vertex exactly once
- Hamiltonian cycle: “path” that visits each vertex exactly once, except that last vertex is same as starting vertex

Example deck

13	11	9	7	5	3	1	2	4	6	8	10	12
12	10	8	6	4	2	13	1	3	5	7	9	11
13	11	9	7	5	3	1	2	4	6	8	10	12
12	10	8	6	4	2	13	1	3	5	7	9	11

- This deck has a Hamiltonian cycle.
- Which decks have a Hamiltonian path but no Hamiltonian cycle?

References

- "Exploring All Binary Mazes", problem 10720, American Mathematical Monthly.
  - Posed 1999 (p. 264) by Knuth
  - Solved 2003 (pp. 60-1) by Lossers and others.
  - "Universal exploration sequences"
- Rubin, F. "A Search Procedure for Hamilton Paths and Circuits." *J. ACM* **21**, 576-580, 1974.