R. Duane Skaggs
Teaching Philosophy

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1 Principles of learning and teaching

A university education should result in a person who is able to think critically. Other skill sets particular to a student's major and choice of career are obviously important, but the primary objective of the university community should be THINKING.

There is often much "pain and anguish" experienced by students while they are learning to think critically. My main responsibility as a faculty member, both inside and outside the classroom, is to help student develop critical thinking abilities. This is true whether the students are in a developmental course or are completing a final senior research project. In the classroom, this goal is accomplished through the use of carefully chosen problems, a mixture of independent and group work, a large amount of Socratic dialogue, and by being available to help students through the frustration that invariably accompanies the learning process.

Obviously it is a cliche to say "The way to learn mathematics (or computer science) is by doing it", but all teachers of these disciplines know this to be true. As a result, I expect my students to put forth the same effort I am putting forth. I will never ask students to perform at a level higher than they are capable, but neither will I allow anyone to perform at a lower level.

I consider learning and teaching in a particular class as a form of partnership that exists on many levels. I have one partnership with each student and another with the class as a whole. However, an equally important partnership exists between members of the class. All three must include, among others, trust, respect, and a level of commitment to the task at hand. If any of these components are missing, the learning experience will suffer.

2 Teaching style

Since my academic career began, I have believed firmly in a "discovery-based" approach to learning. I often use a sequence of questions to lead students toward certain desired conclusions, but I also utilize many open-ended questions that have many possible outcomes. These questions develop thinking skills and provide the opportunity for healthy discourse regarding appropriate or acceptable responses to questions.

As part of the discovery process, technology can play an important role. A well-constructed simulation, a graph of a function, or the results of some exploratory data analysis may allow students to see the key to the solution of a problem. For example, many people had thought Euclidean geometry held few open problems but interactive geometry software has revealed many new and exciting results. Recently I also have begun to use Blackboard in teaching classes. This use of technology, especially options such as discussion boards, has proved to be quite popular among my students.

I am an active supporter of collaborative work by students and assign many group assignments during the course of a term. Depending on the particular class, all group work may be completed during the meeting time or some may be assigned as take-home projects. In many classes, I also use a "learning to think by writing" approach in which selected problems are completed then undergo peer review before the final solutions are graded. Improved communication skills, both verbal and oral, are developed in these sessions.

3 Polya's Four Steps of Problem Solving

In his landmark book *How to Solve It*, the incomparable and influential mathematician and teacher George Polya presented four main steps of problem solving. At first glance, these steps seem like "common sense" but careful practice can allow students to become expert problem solvers.

1. Understand the Problem
2. Devise a Plan
3. Execute the Plan
4. Look Back and Around

Regardless of the class - lower level, upper level, mathematics, computer science - I begin with a discussion of Polya's steps to problem solving, how they are useful, and how rarely most people use them. I allow the students to define the term "problem" and with a little prodding the definition looks something like the following:

> A problem is a situation that seems should have some kind of answer but we do not know how to reach it.

Therefore, what is a problem to one person may not be a problem to someone else. This definition, or one similar, rules out "$1 + 1 = ?$" as a problem in a college classroom. It does not, however, rule out the situation of 500 such exercises to be done in 30 seconds.

When presented with a problem, most people jump to the third step and immediately begin to do something. If situations are familiar enough then this is may not be troublesome, but in an unknown situation this can have disastrous effects. It is just this type of scenario we focus on for at least the first day of every new term, and a point to which we return often. Through the term, students begin to realize that each step is of equal importance and that all must be completed in order to become a good problem solver.

4 A typical introduction problem

Many problems that I use to develop critical thinking skills are deceptively simple. For example, consider the following Polya problem that I have used for several years with much success.

> Can you cut a pizza into 11 pieces using exactly four straight cuts?

Experience shows that most students begin by drawing a circle and drawing lines to represent slices. The first few attempts yield eight, then nine pieces. After a few more trials most students get 10, then stop and say there is no way to get 11. By asking them to look at how they went from eight pieces to nine pieces, then from nine to 10, everyone eventually sees how to get the 11 pieces requested. I then ask whether every pizza is round, and whether it matters. There is usually some grumbling, but most of the students start working immediately on square and rectangular drawings. Someone draws a star-shaped pizza, or I mention the heart-shaped pizza one restaurant sells on Valentine's Day, and suddenly four cuts are able to get considerably more than 11 pieces. By this time, most of the class is typically drawing wilder and wilder shapes to represent the pizzas.

I then ask the class to go back to the original round pizza and ask whether they think they can get more than 11 pieces with exactly four straight cuts on that shape. Most students by now think that at least one "indented" side is needed, so they are quick to deny the possibility. Invariably, though, someone mentions that there is no need to cut straight down on the pizza. Why not use one of the cuts to cut off the toppings? If the toppings are not removed, the second and third cuts can produce eight pieces rather than the seven that seemed to be the maximum before. Then folding the pizza is suggested, and the discussion continues to grow.

Depending on the particular class, we may spend an hour exploring the different possibilities. A simple question originally answered with a circle and four line segments drawn on a piece of paper can turn into a discussion on topics ranging from exponential growth (What happens if you keep folding the pizza in half?) to topological equivalence (What if we used a "circular" cut instead?) to various concepts of proof (How would we restate the problem to allow only traditional pizza cuts? Could we then convince others that 11 pieces is the maximum possible with four straight cuts?).

By using a combination of open-ended questions such as this one and some more closed-form questions, the students develop the types of skills that improve all of their problem solving abilities, without reaching the level of frustration that causes them to simply give up. The questions I use are undeniably difficult, but they are presented in such a way that many students tell of holiday family discussions about the problems, and several have asked for additional questions after the class is over so they can continue to share with their families.

5 Undergraduate research

As part of the "learning by doing" paradigm, it is crucial that undergraduates participate in at least one prolonged research project before graduation. I suggest that students research areas which will allow them to achieve new results; this can definitely add to or develop a sense of confidence. Relatively young fields such as graph theory have numerous unexplored areas in which undergraduates can obtain new results. Other areas that I typically suggest for potential research include combinatorial games and experimental studies such as software-based geometry explorations.

Name	Project title
Michael McCarty*	*A 3D model of planetary interiors*
Shannon Hord	*Information hiding through digital steganography*
Michael McCarty	*Automated theorem proving and natural language processing*
Kevin Swartz	*Vertex labels and label sums of paths and cycles*
Adam Eldridge*	*The non-attacking queen's problem*
Michael Slone	*Model theory and Smullyan systems*
Michael Howard	*Applications of domination in graphs*
May Hylton	*Random knots and graphs*
Michael Howard	*Knots, graphs, and applications*
James Hannah	*Implementation of an algorithm to find Groebner bases*
John Liston*	*Applications of linear algebra to data compression*
Jacki Pettigrew*	*Quadratic programming and portfolio management*

Table 1: Senior research projects

I served as co-sponsor for the projects marked with an asterisk. In addition, I have supervised three group research projects, entitled *Paradoxes, fallacies, and philosophies of mathematics*, *Paradoxes in social choice* and *Is Vegas-style Solitaire fair?*.

6 An early influence

Some of the best teachers I ever had used techniques similar to those described by Polya. In some cases I did not immediately recognize the quality of the teachers, but as I matured I began to appreciate the thinking skills they had developed. My high school history teacher, Betty Beaumont, forced me to think more deeply than had any of my previous teachers. For instance, she once asked if we *really* believed that American colonists would actually dump extremely expensive tea into Boston Harbor. At the time, I understood her to say "The Boston Tea Party did not happen" when in fact she was prodding us to explore fully the reasons behind the event on our own rather than to simply accept the story we were told. This lesson has stayed with me for nearly fifteen years, and has had a profound influence on my teaching.